REŠEVANJE DIFERENCIALNIH ENAČB Z MEHANSKIMI RAČUNSKIMI STROJI

Pino Koc

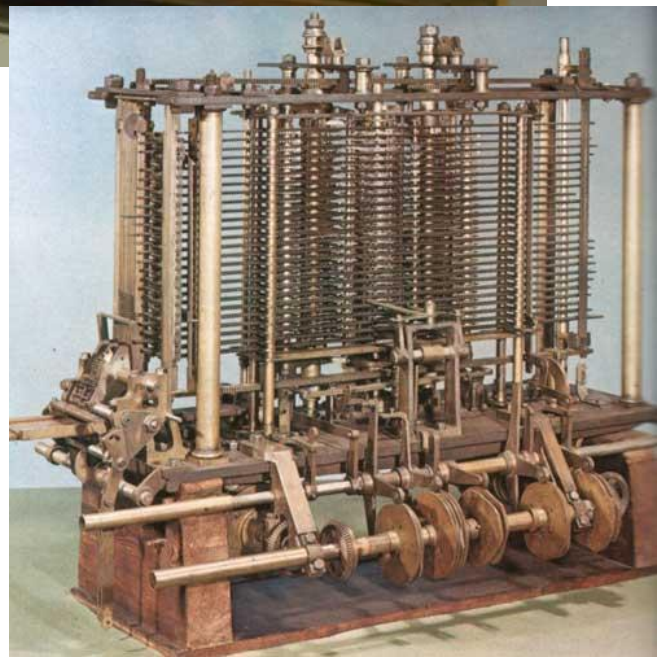
Seminar za učitelje matematike

FMF, Ljubljana, 25. september 2015

Vir: [1]

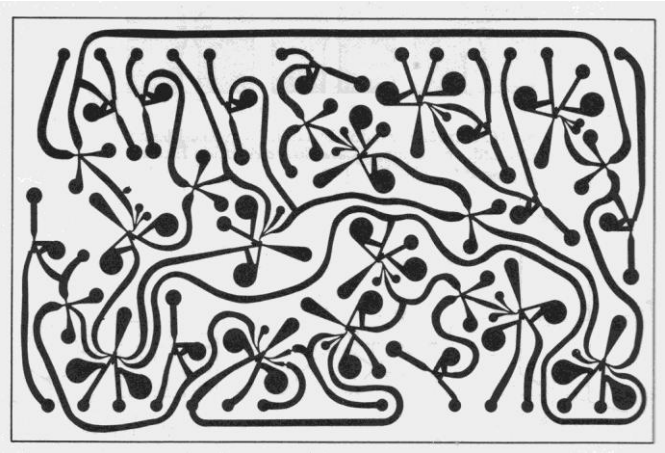
Nekateri pripomočki in naprave za računanje:

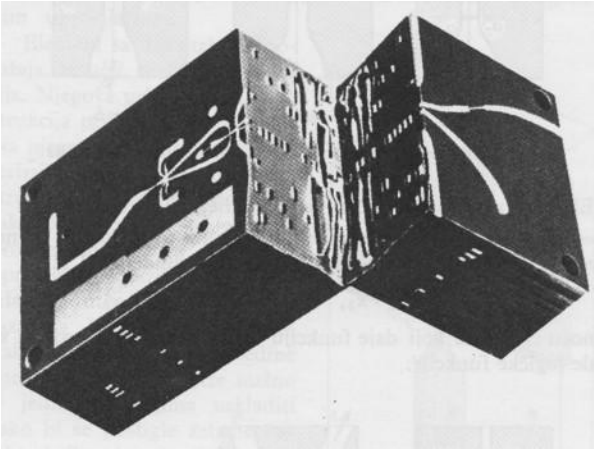
1a) Digitalni (mehanski)



Vir: http://history-computer.com/ , [2]

1b) Digitalni (fluidni)



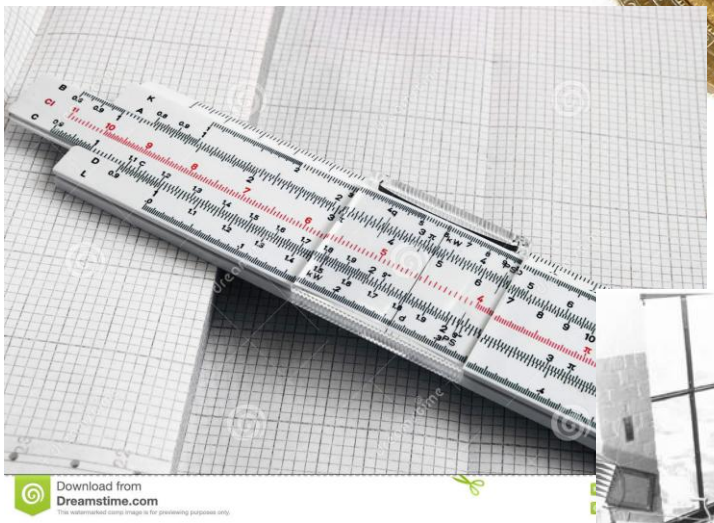


Vir: [3]

1c) Digitalni (relejni)

1d) Digitalni (elektronski)

2) Analogni (mehanski)



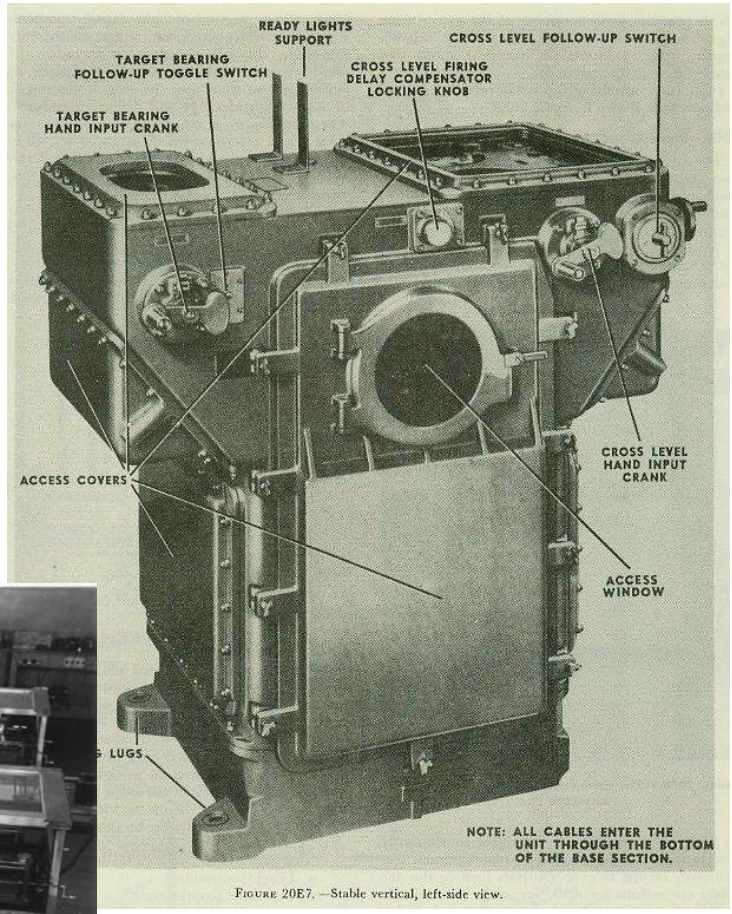
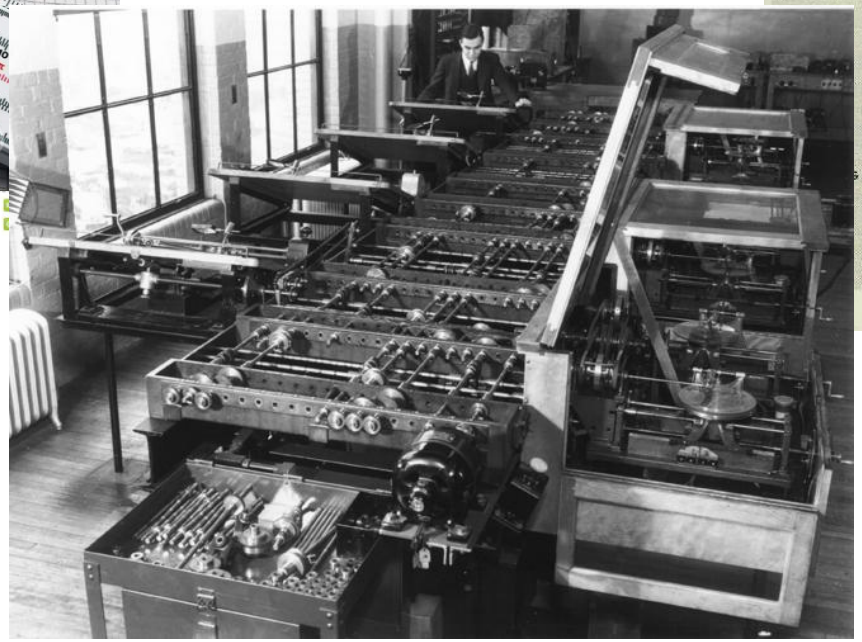


FIGURE 20E7. —Stable vertical, left-side view.



Lastnosti mehanskih analognih računskih strojev

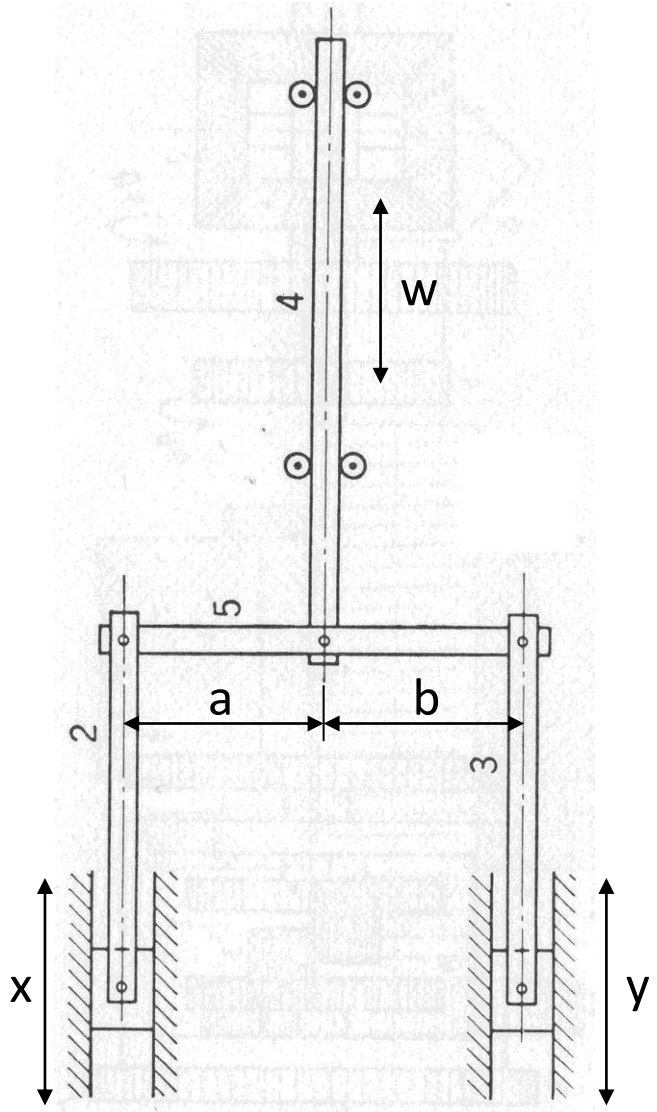
- **osnovne spremenljivke so pomiki in/ali zasuki,**
- **zvezno spreminjanje količin,**
- **mehanski principi delovanja komponent stroja so ekvivalentni računskim operacijam, ki jih te komponente izvajajo,**
- **zmožnosti izvajanja seštevanja, odštevanja, množenja, deljenja, integracije, razstavljanja vektorjev na komponente, računanja s trigonometričnimi funkcijami, računanja s poljubnimi, »hardwersko« nastavljenimi funkcijami.**

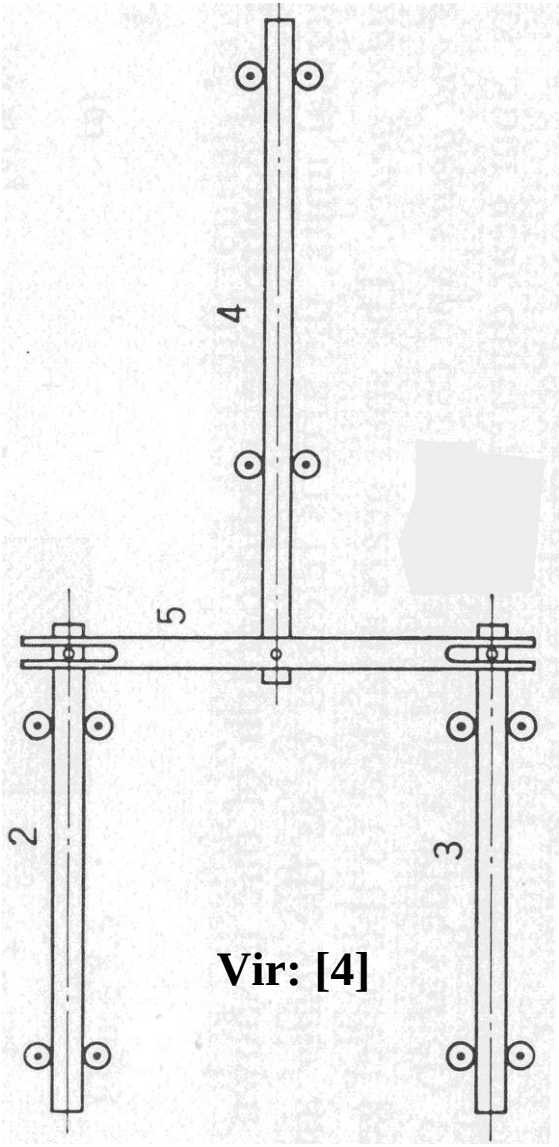
- **omejena natančnost,**
- **težavno programiranje stroja,**
- **počasnost, obsežnost, energijska potratnost.**

Računske operacije: (1) Množenje s konstanto pri translaciji

Postavimo: w = 0

$$\frac{x}{a} = \frac{y}{b} \quad \rightarrow y = \frac{b}{a}x$$





Vir: [4]

Napake pri znatnih zasukih droga 5.

Računske operacije: (1) Množenje s konstanto pri rotaciji

$$dl = r_A\, d\varphi_A = r_B\, d\varphi_B$$

$$\varphi_B = \frac{r_A}{r_B}\, \varphi_A$$

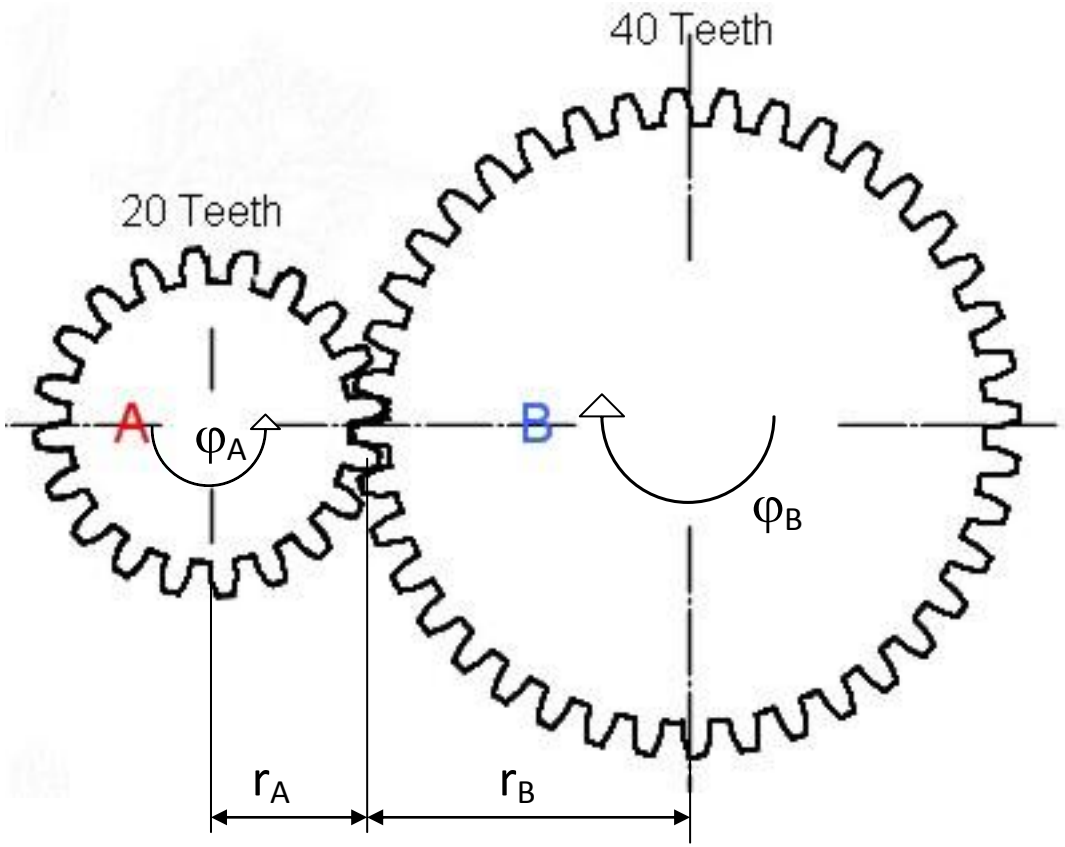
ker

$$ob_A = 2\pi\, r_A = m\, z_A$$

$$ob_B = 2\pi\, r_B = m\, z_B$$

je

$$\frac{r_A}{r_B} = \frac{z_A}{z_B}$$



Računske operacije: (2) Seštevanje, odštevanje pomikov

Najprej postavimo: x = 0

$$\frac{w_y}{a} = \frac{y}{a+b} \quad \rightarrow w_y = \frac{a}{a+b} y$$

nato: y = 0

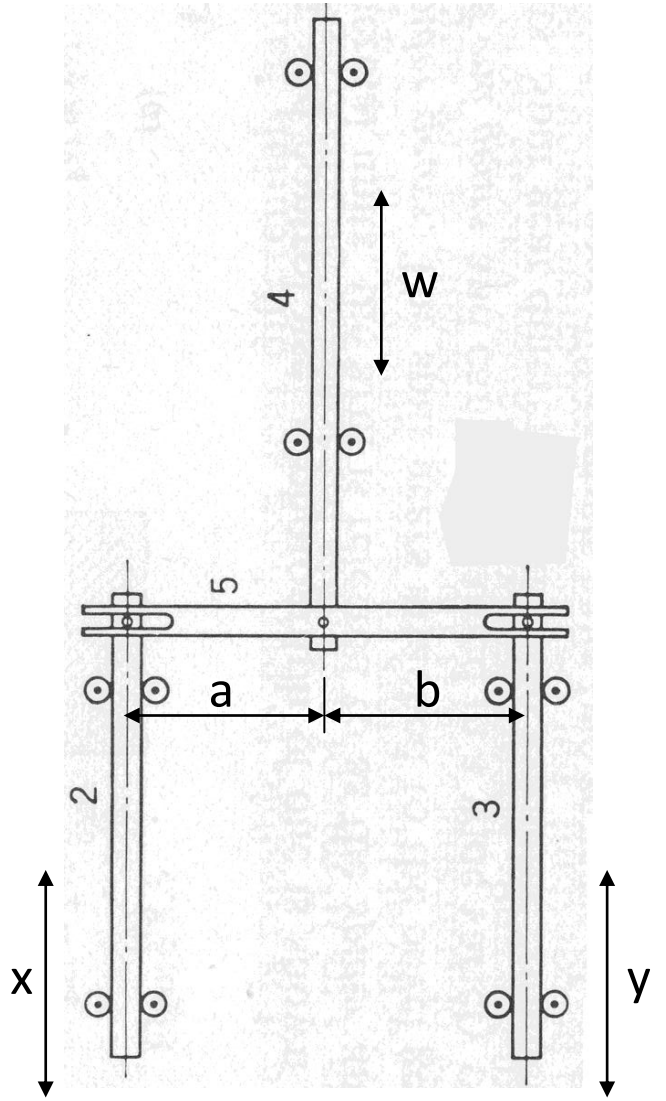
$$\frac{w_x}{b} = \frac{x}{a+b} \quad \rightarrow w_x = \frac{b}{a+b} x$$

Seštejemo:

$$w_x + w_y = w = \frac{bx+ay}{a+b}$$

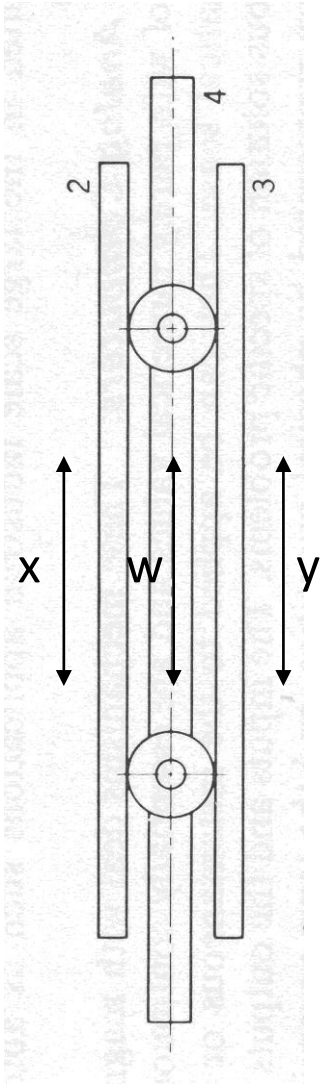
Običajno: a=b

$$w = \frac{x+y}{2}$$



Računske operacije: (2) Seštevanje, odštevanje zasukov

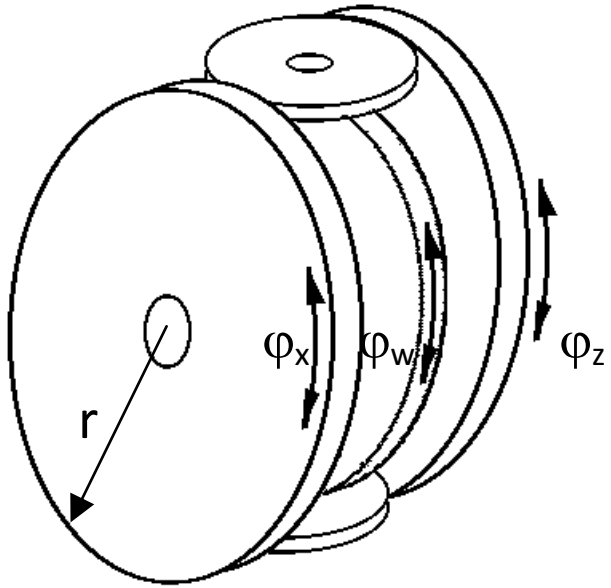
Diferencial



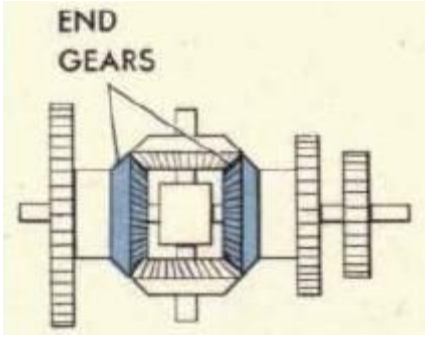
$$w = \frac{x+y}{2}$$

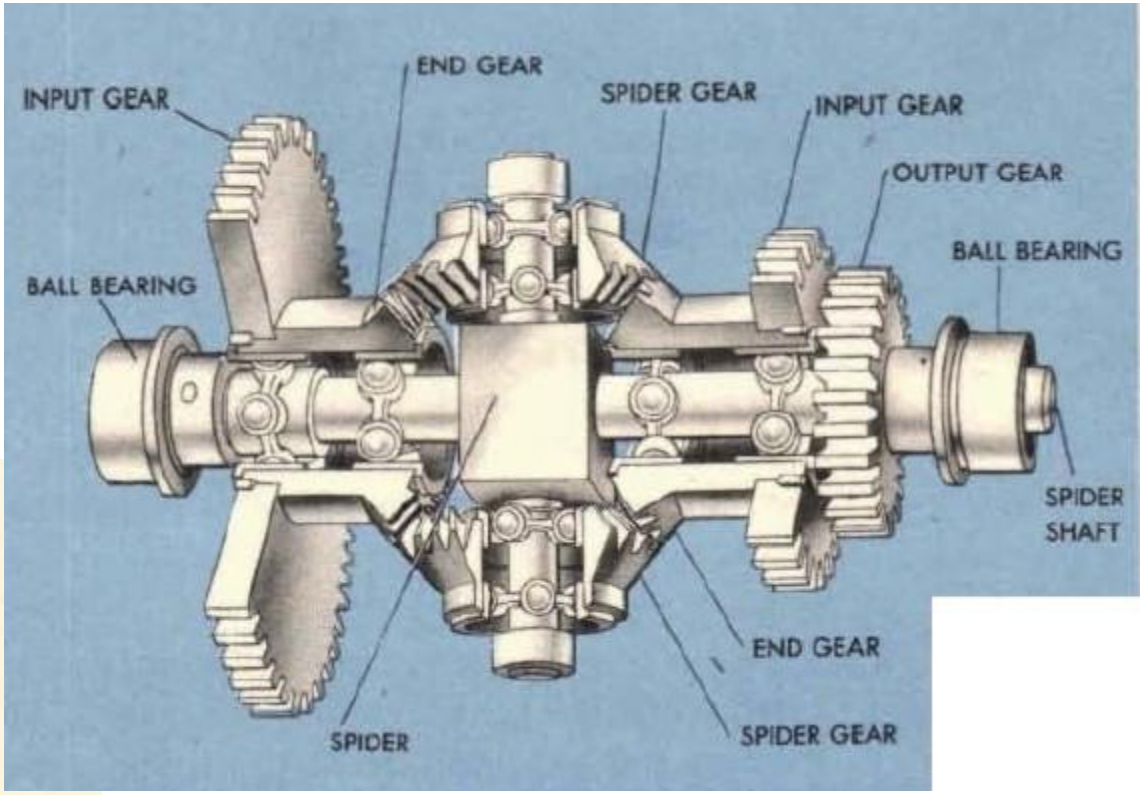
$$x = r\,\varphi_x\,, \quad w = r\,\varphi_w\,, \quad y = r\,\varphi_y$$

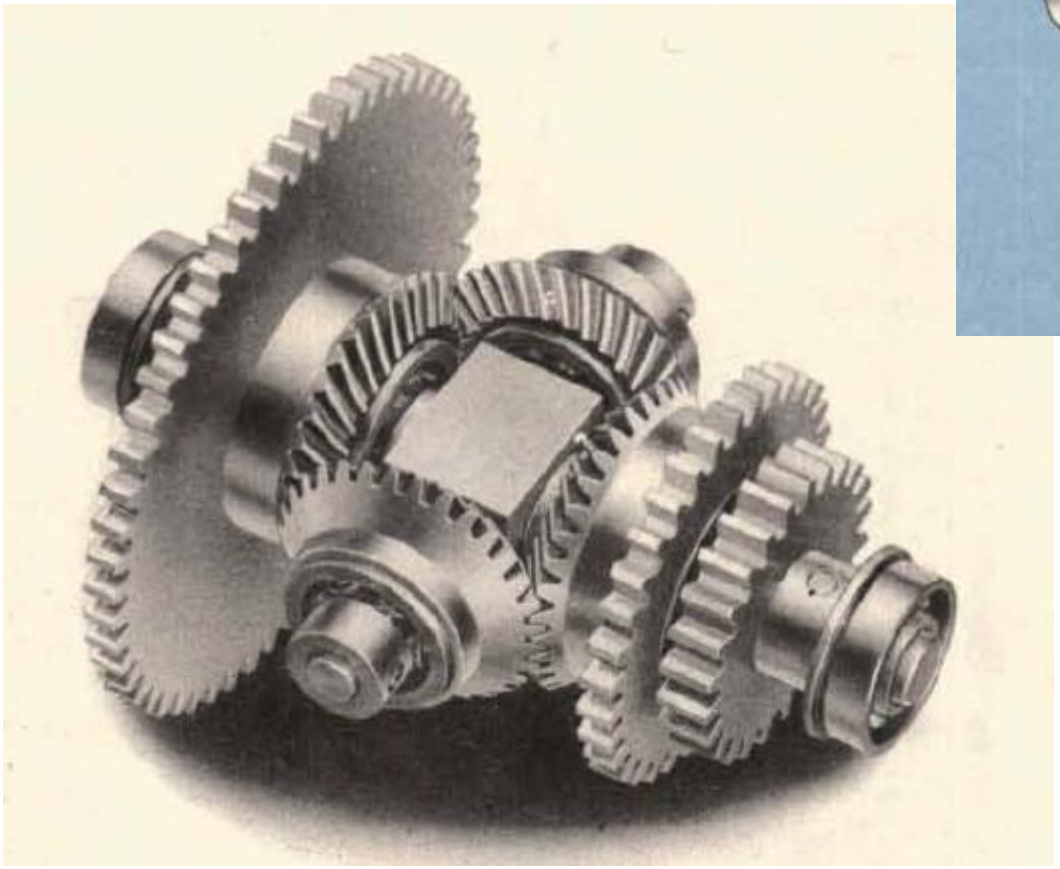
$$\varphi_w = \frac{\varphi_x+\varphi_y}{2}$$



Diferencial v računskih strojih



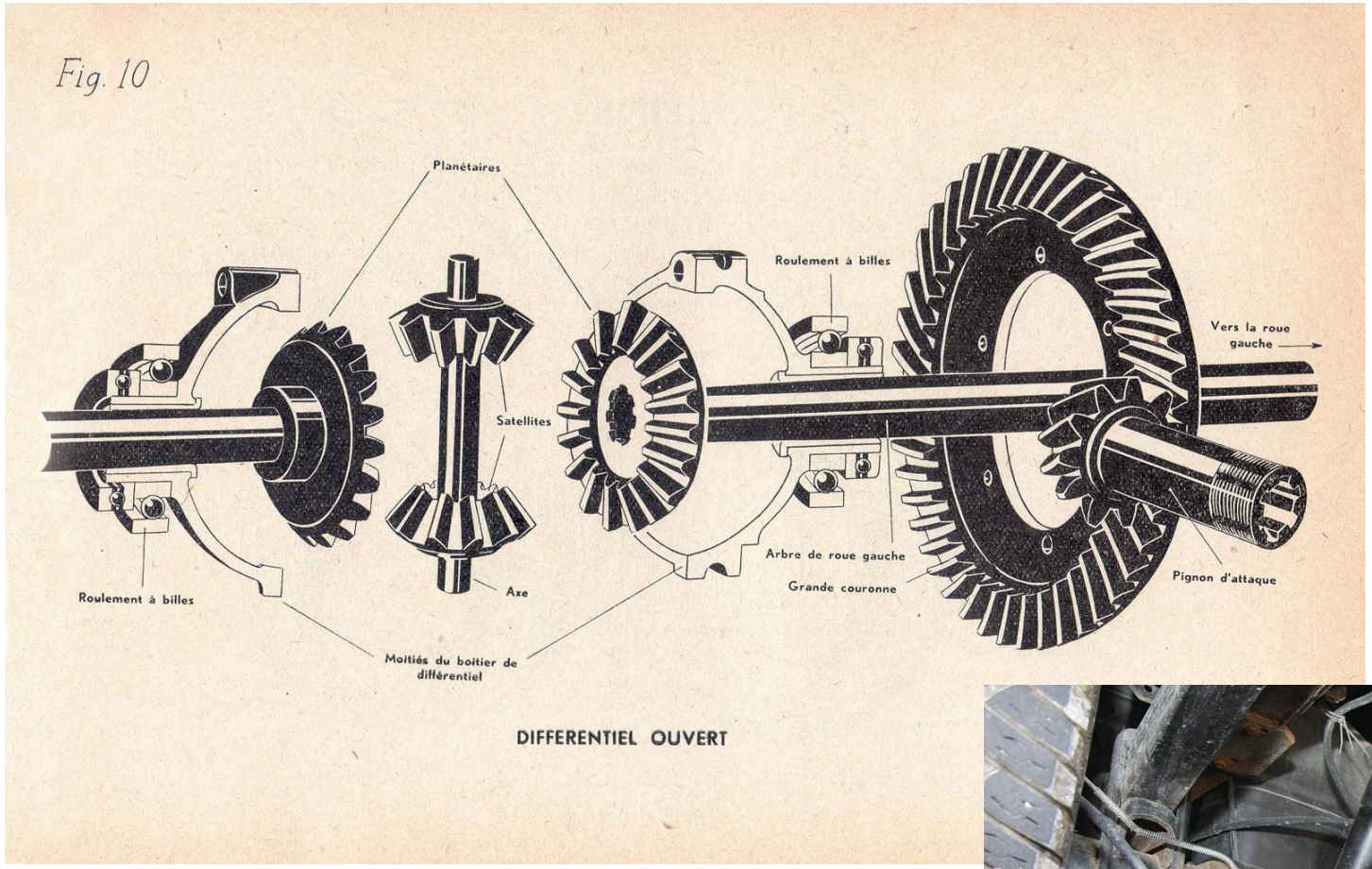




Natančnost 5 do 10 kotnih minut.

Vir: [5]

Diferencial v avtomobilih



Računske operacije: (3) Integracija

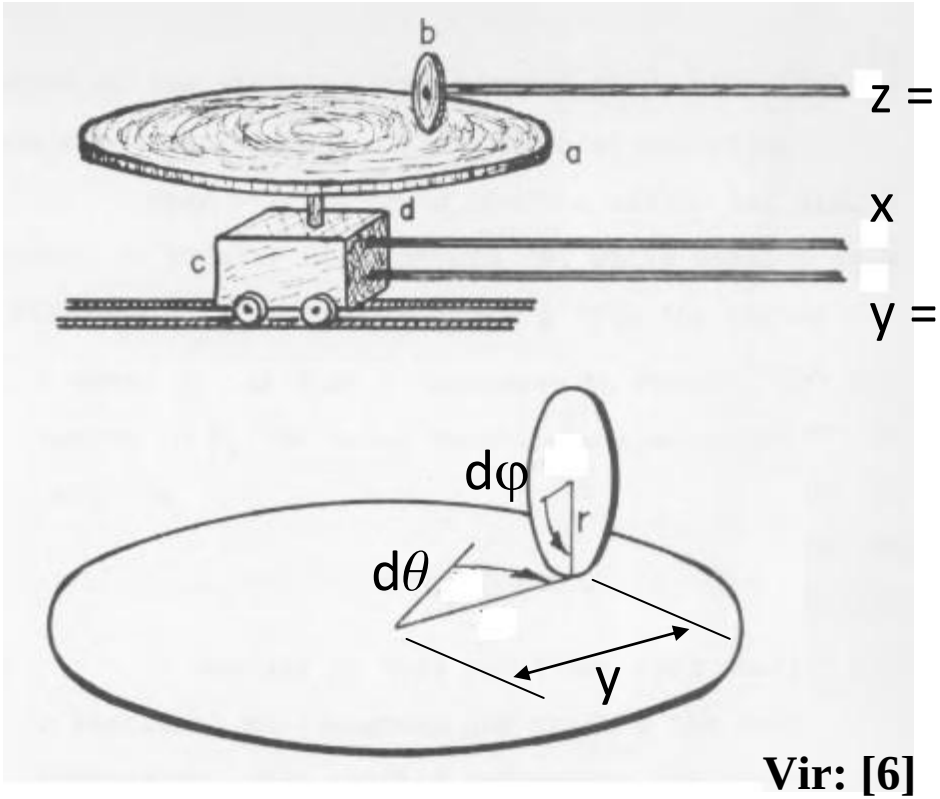
Integrator z diskom

$$y\,d\theta = r\,d\varphi$$

$$\varphi = \frac{1}{r}\int y\,d\theta$$

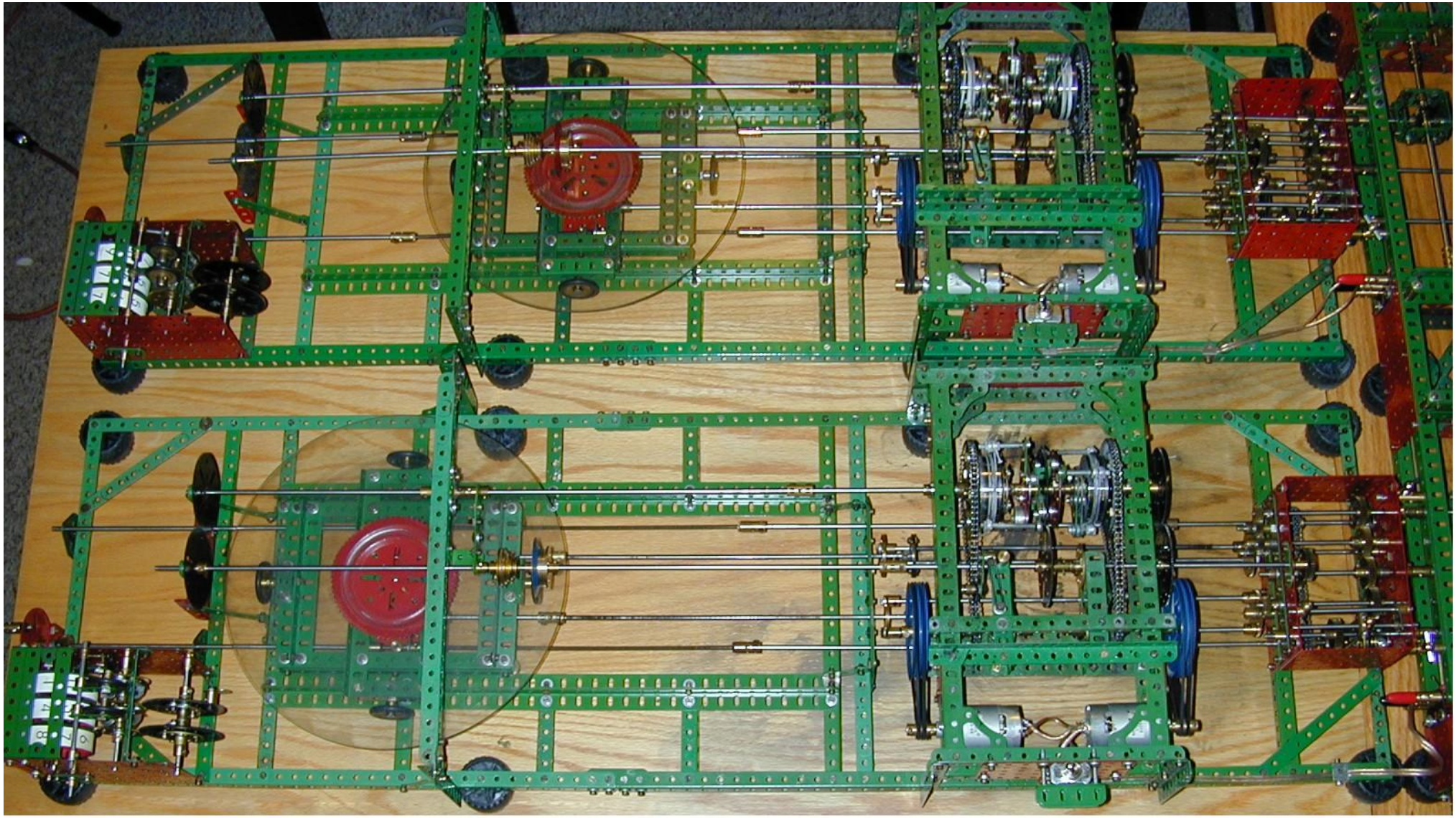
$$\varphi \to z\,,\theta \to x$$

$$z = \frac{1}{r}\int y\,dx$$



Vir: [6]

Integrator z diskom, izvedba

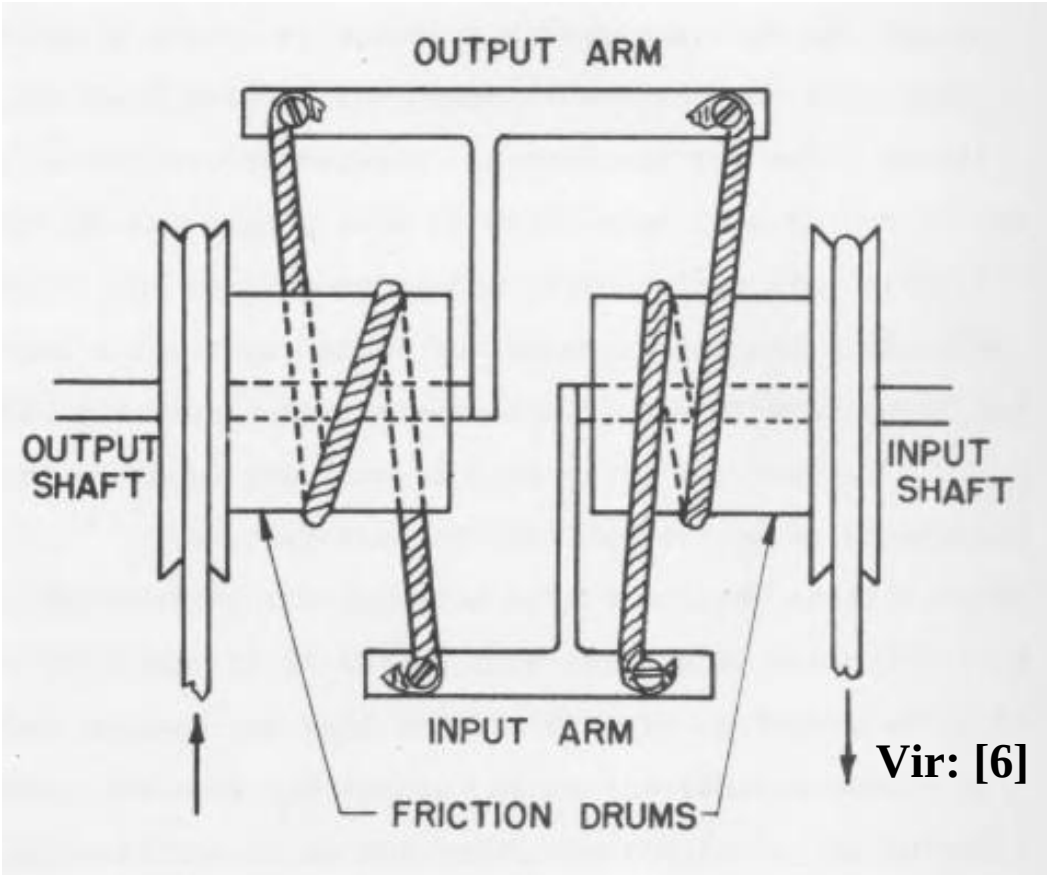


http://www.meccano.us/differential_analyzers/robinson_da/

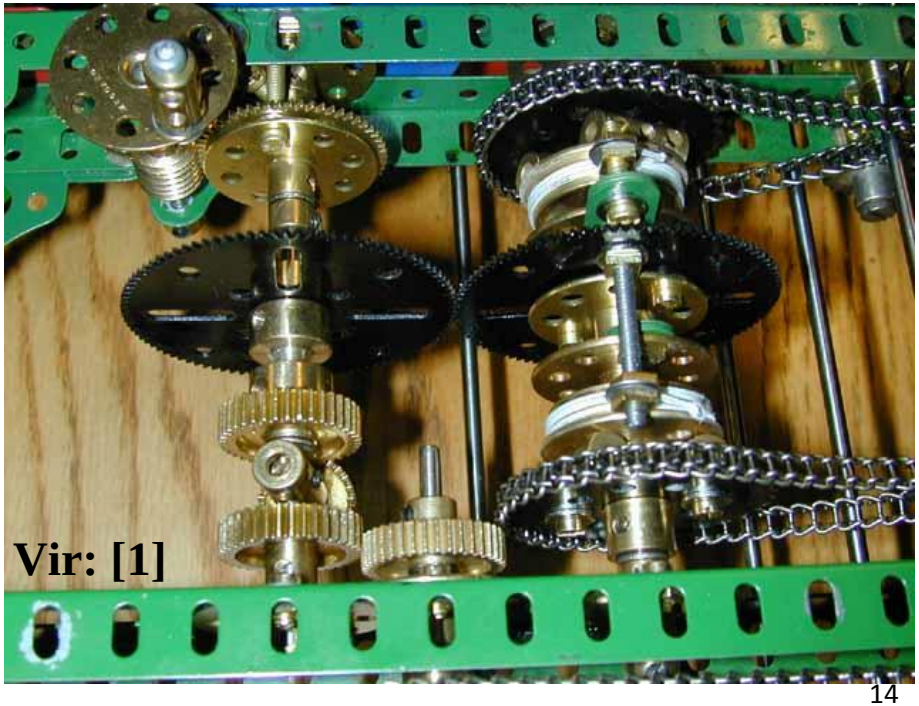
Vir: [1]

Ojačevalnik torzijskega momenta

$$F_{vitla} = F_{roke}\, e^{\mu \hat{\alpha}}$$



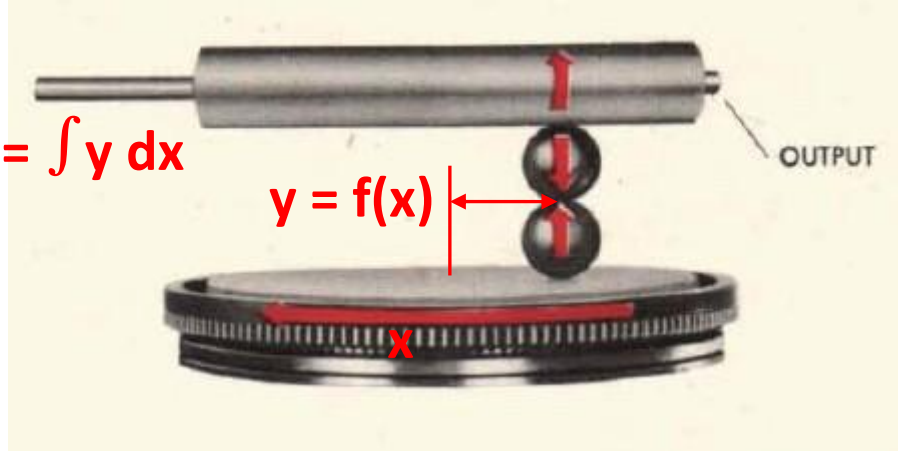
Vir: [6]



Vir: [1]

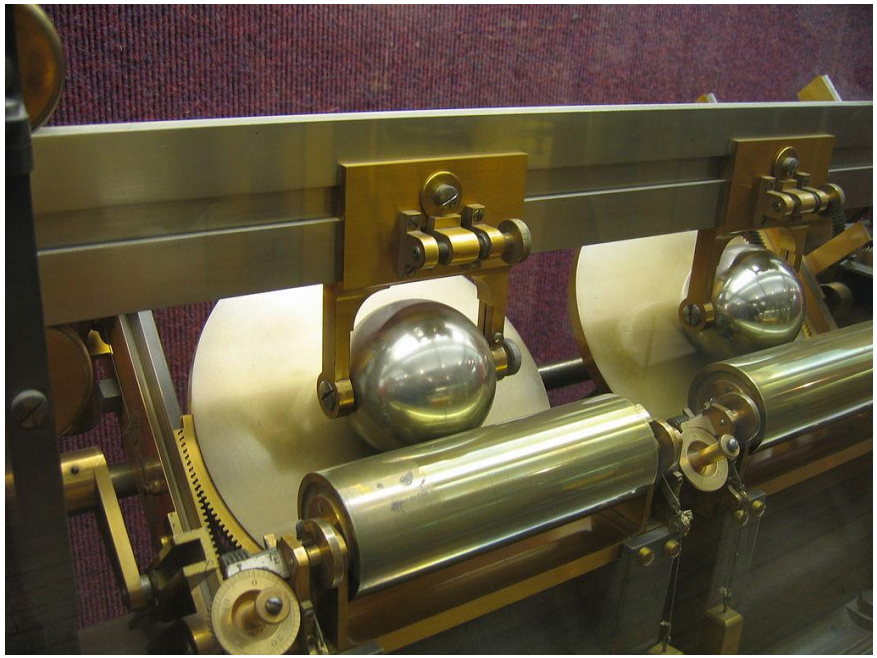
Računske operacije: (3) Integracija

Integrator z valjem

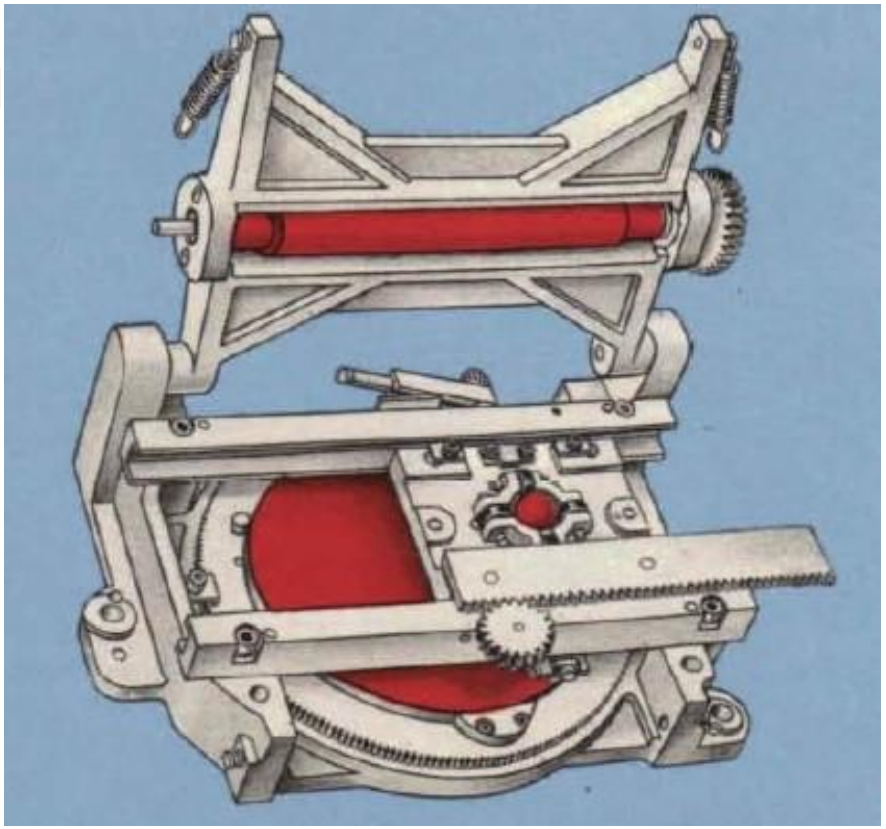


Natančnost 0,01% do 0,5%.

Vir: [7]



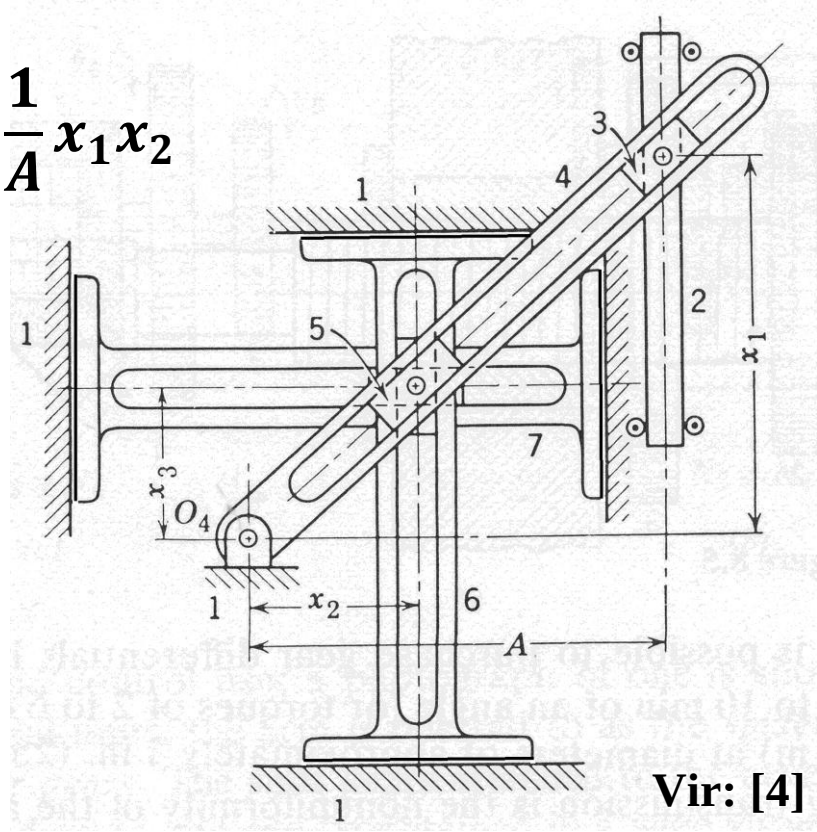
Vir: [5]



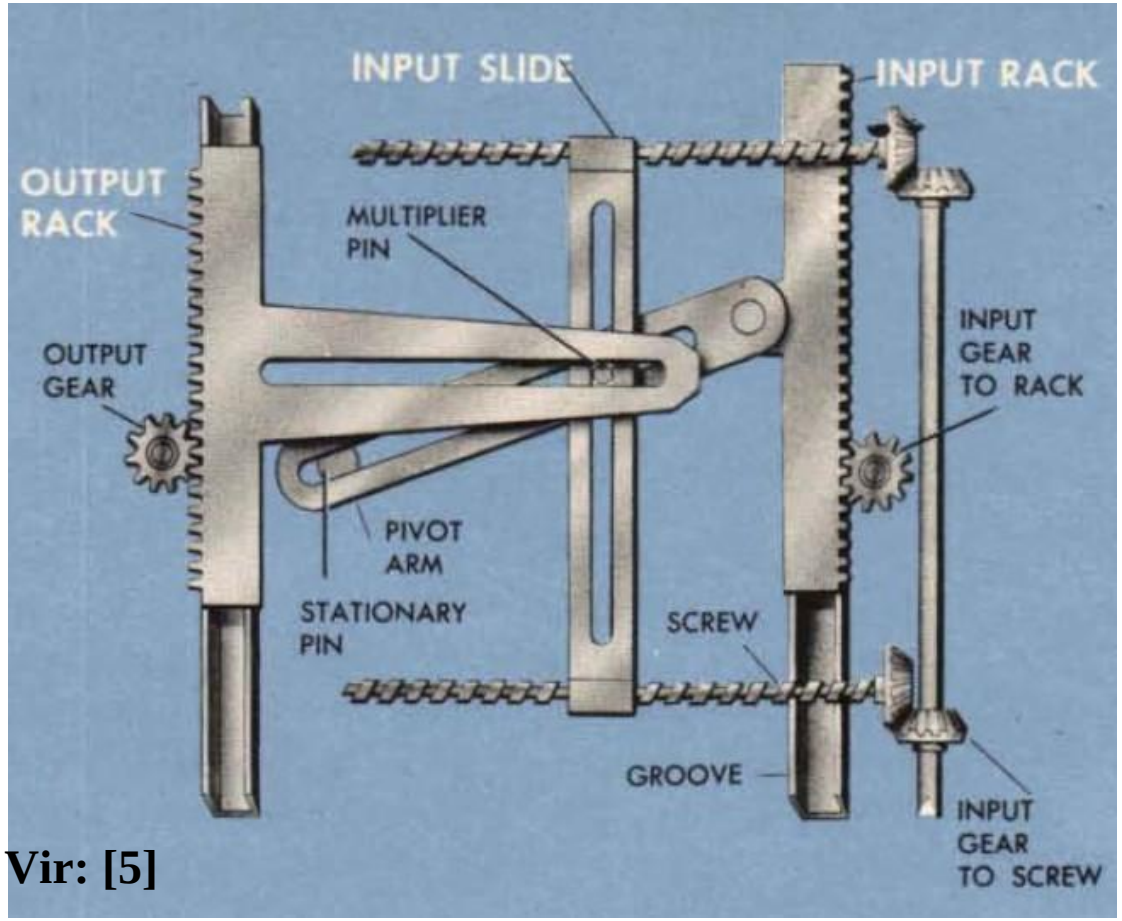
Računske operacije: (4) Množenje, deljenje

Množenje pomikov

$$\frac{x_1}{x_3} = \frac{A}{x_2} \quad \rightarrow x_3 = \frac{1}{A} x_1 x_2$$



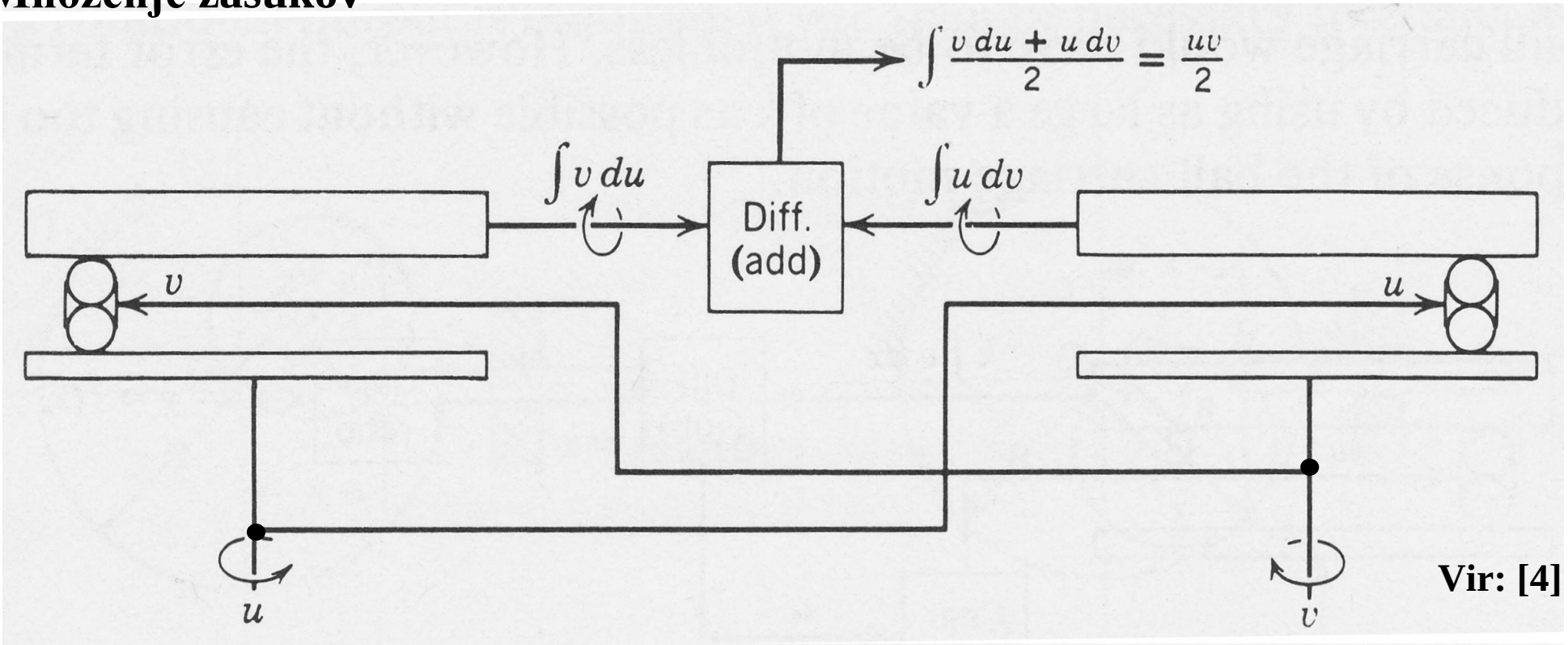
Vir: [4]



Vir: [5]

Računske operacije: (4) Množenje, deljenje

Množenje zasukov



Vir: [4]

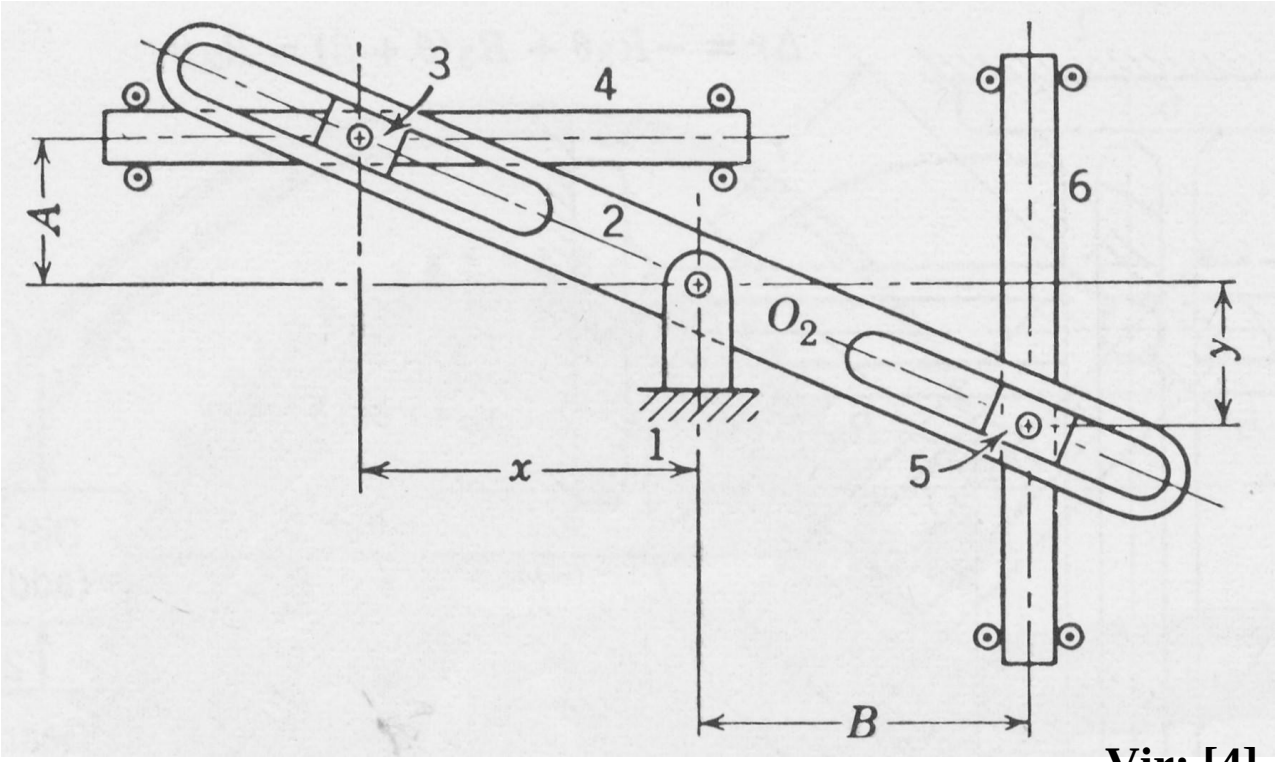
Deljenje:

$$\frac{x}{y} \rightarrow x\ \frac{1}{y}$$

Računske operacije: (5) Invertiranje

Recipročne vrednosti pri pomikih

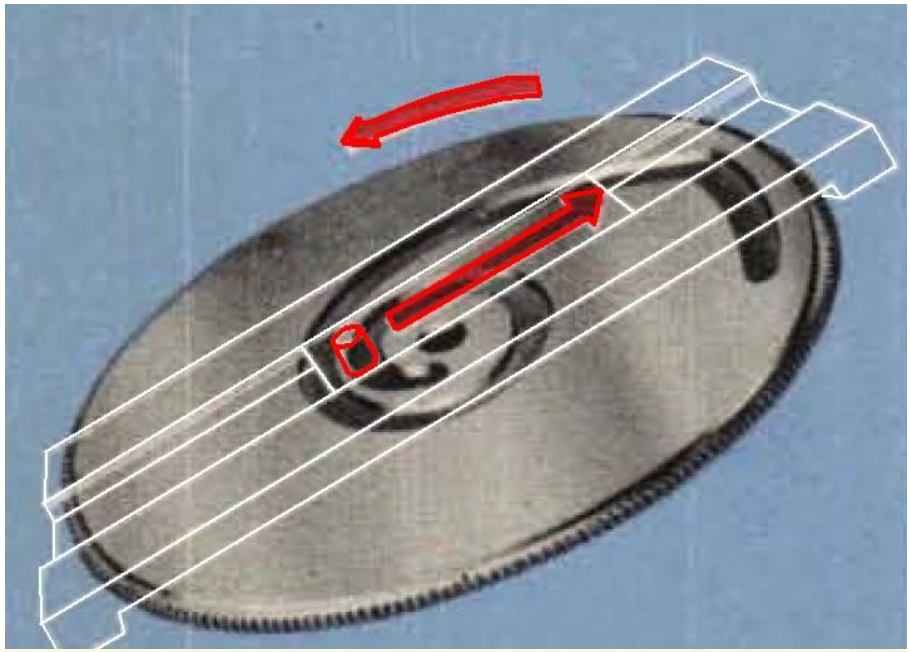
$$\frac{x}{A} = \frac{B}{y} \quad \rightarrow y = AB\frac{1}{x}$$

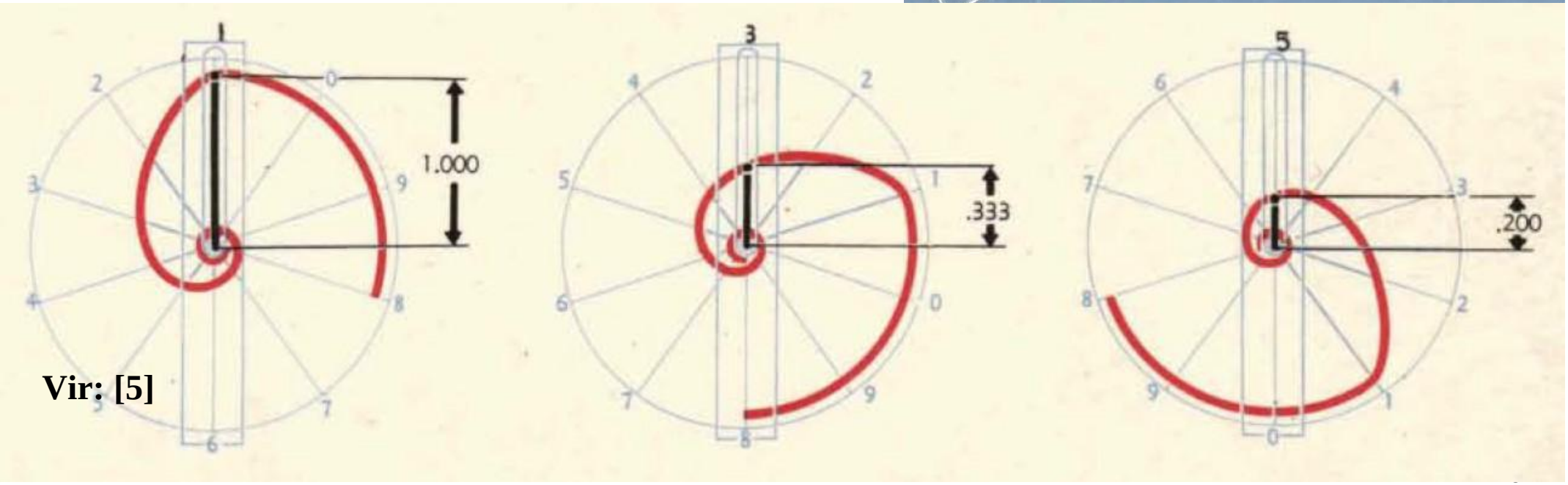


Vir: [4]

Računske operacije: (5) Invertiranje

Recipročne vrednosti pri zasukih

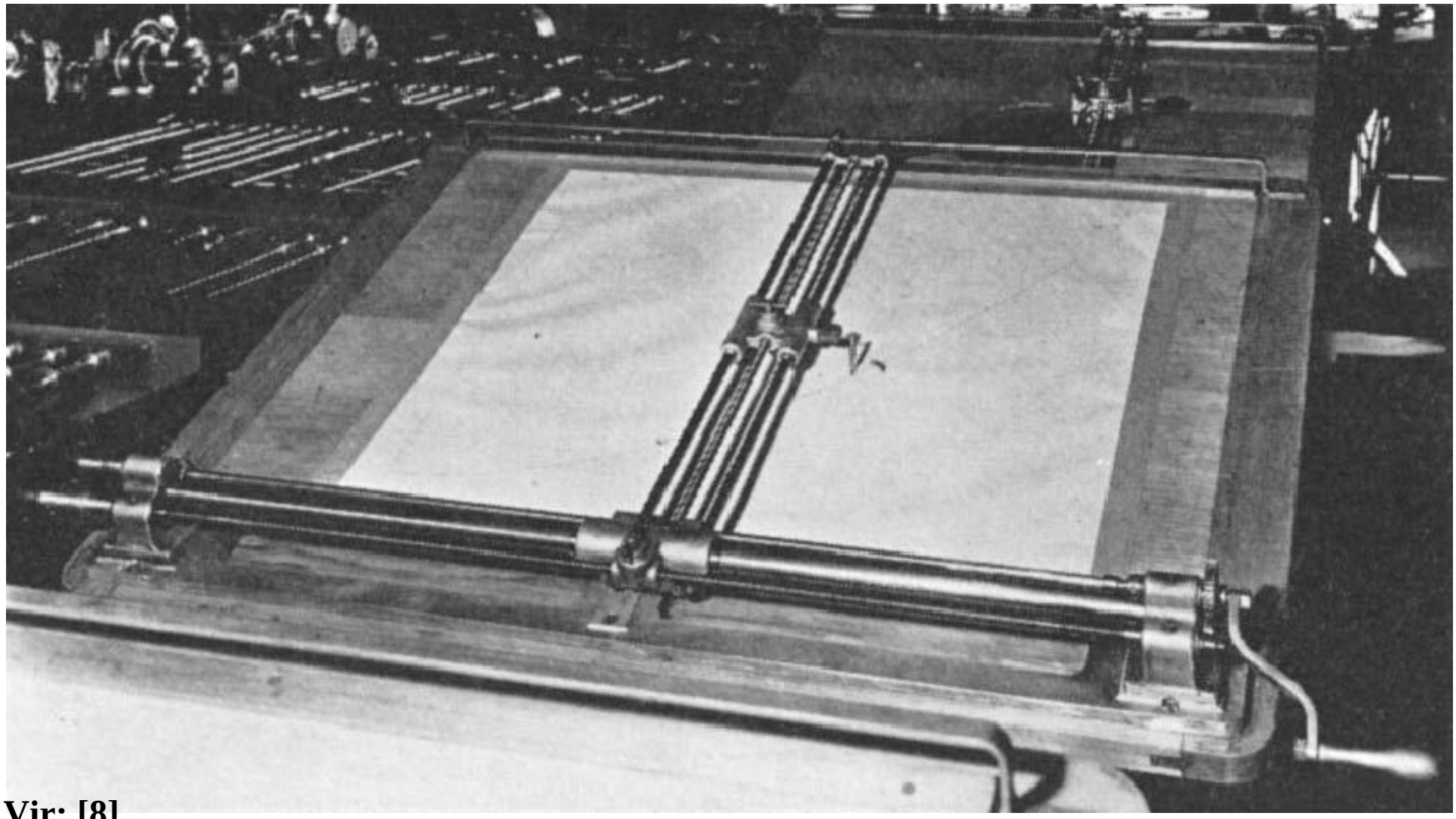




Vir: [5]

Računske operacije: (6) Generiranje funkcij

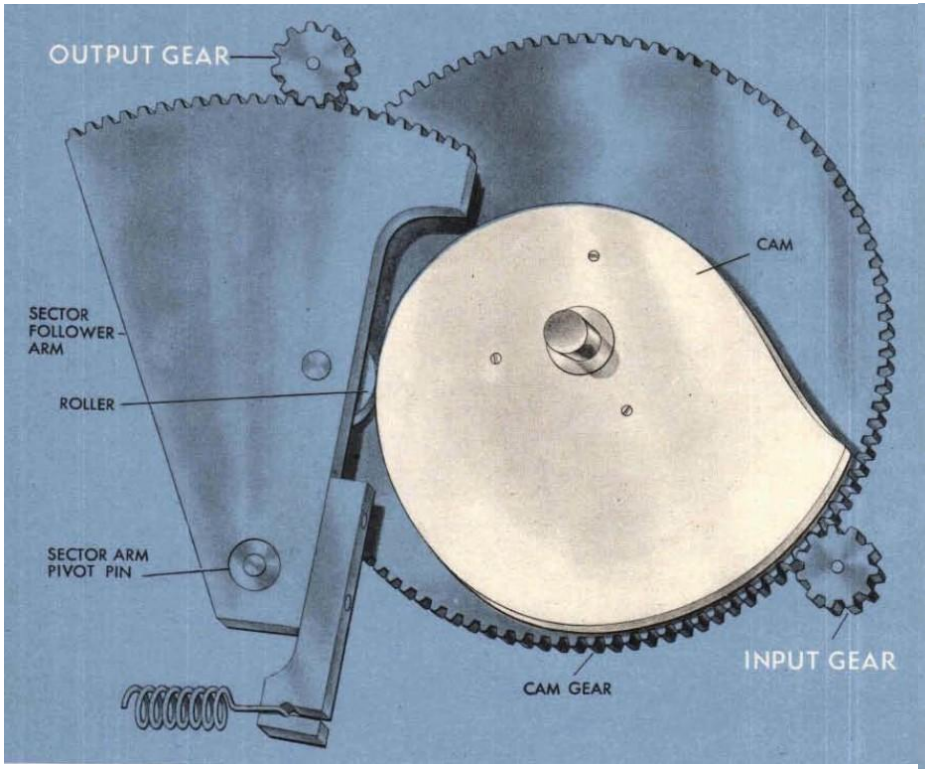
Tabla za vnos krivulje

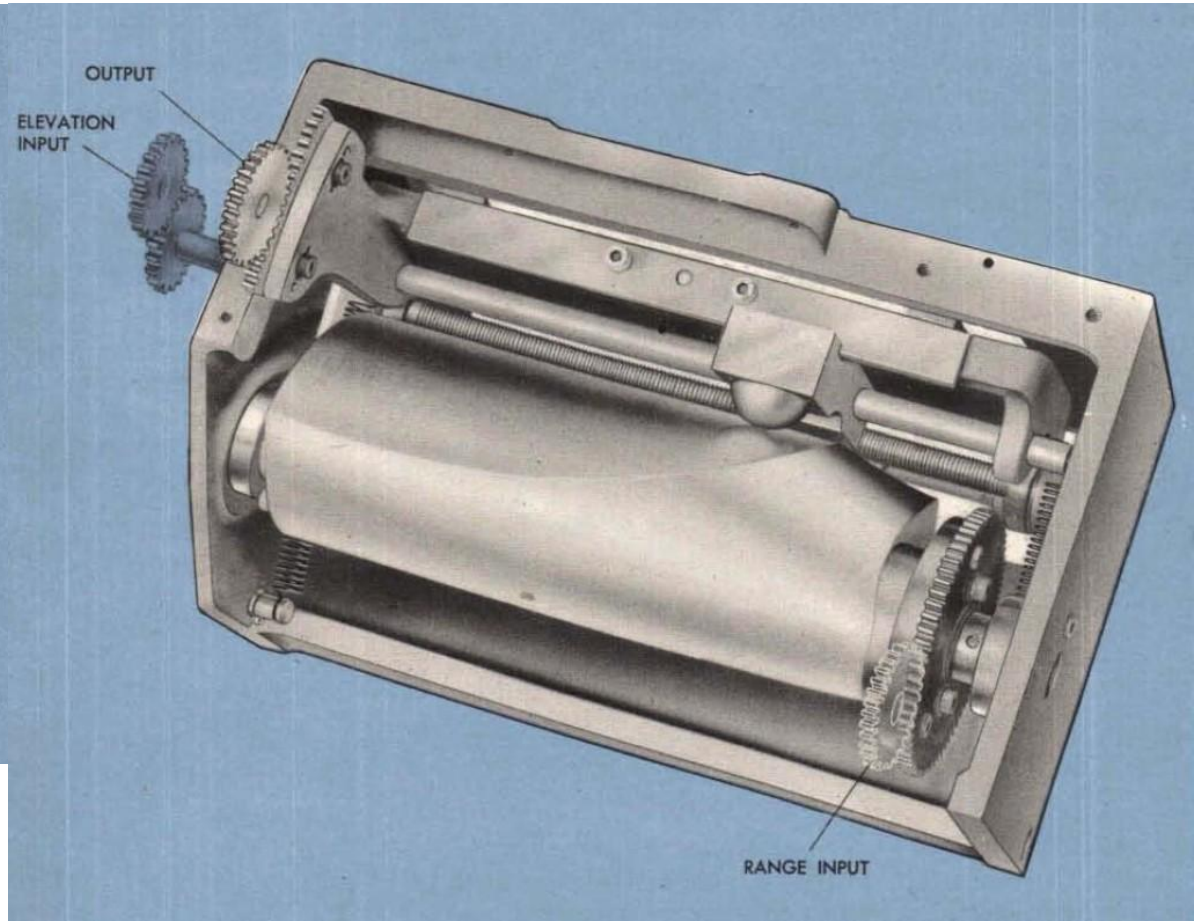


Vir: [8]

Računske operacije: (6) Generiranje funkcij

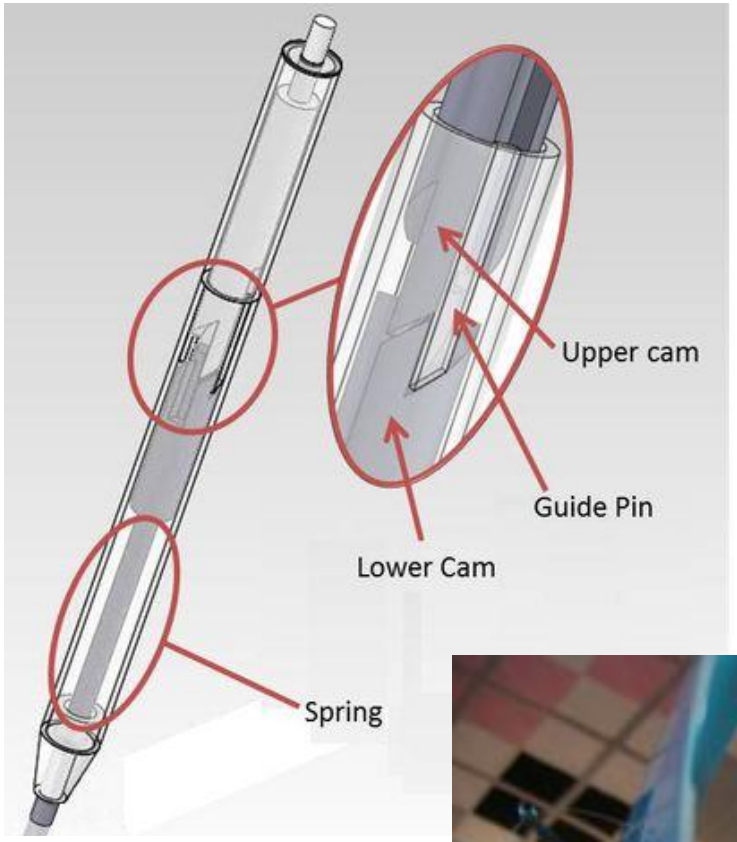
Krivuljni mehanizmi v računskih strojih

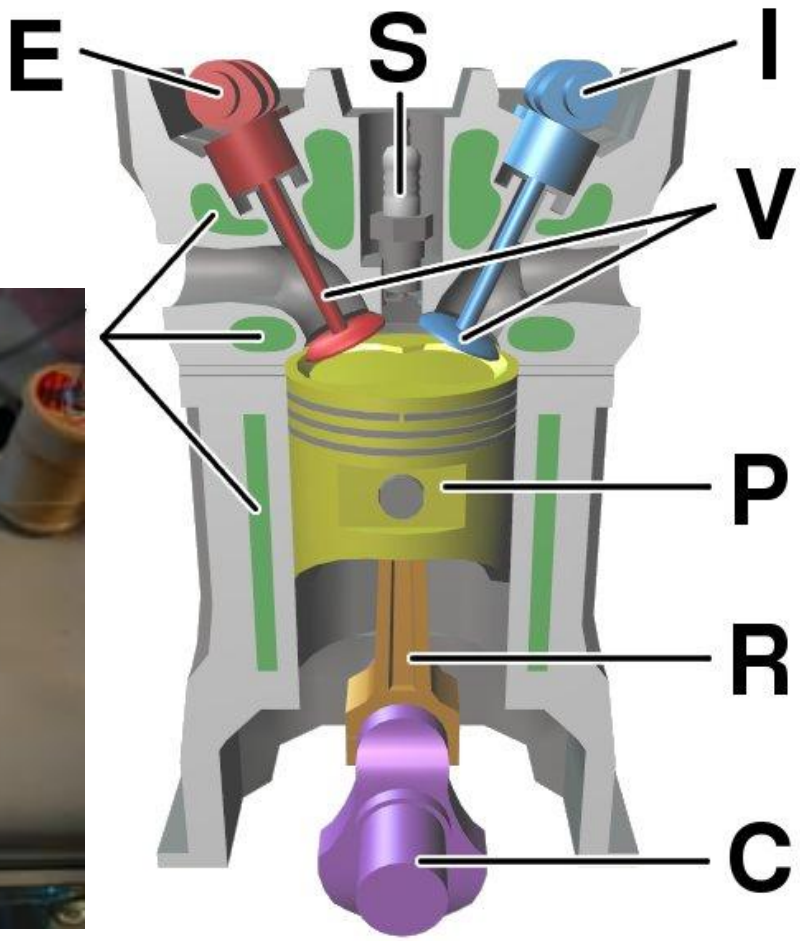




Vir: [5]

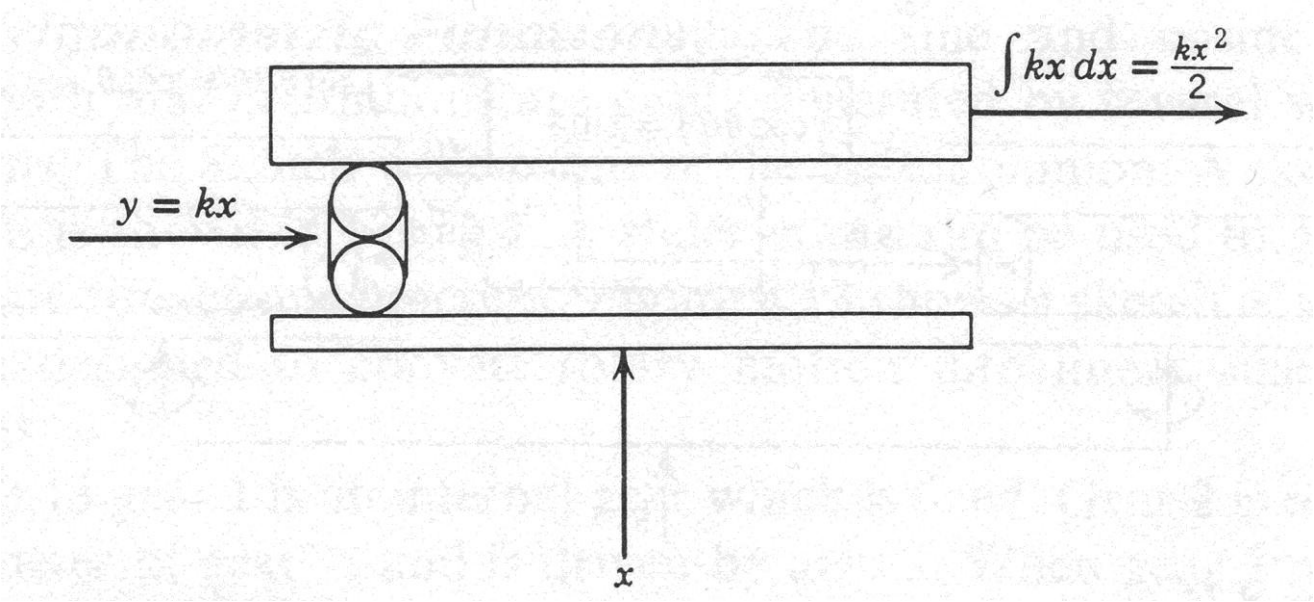
Krivuljni mehanizmi v našem vsakdanu

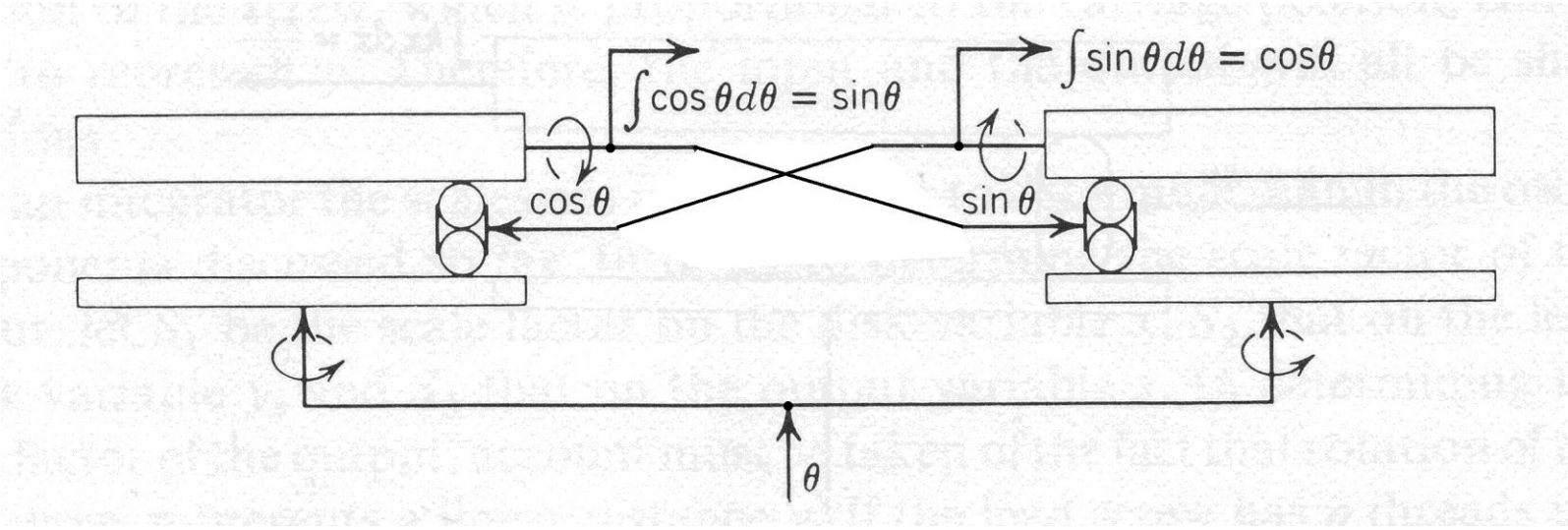




Računske operacije: (6) Generiranje funkcij

Z uporabo integratorjev

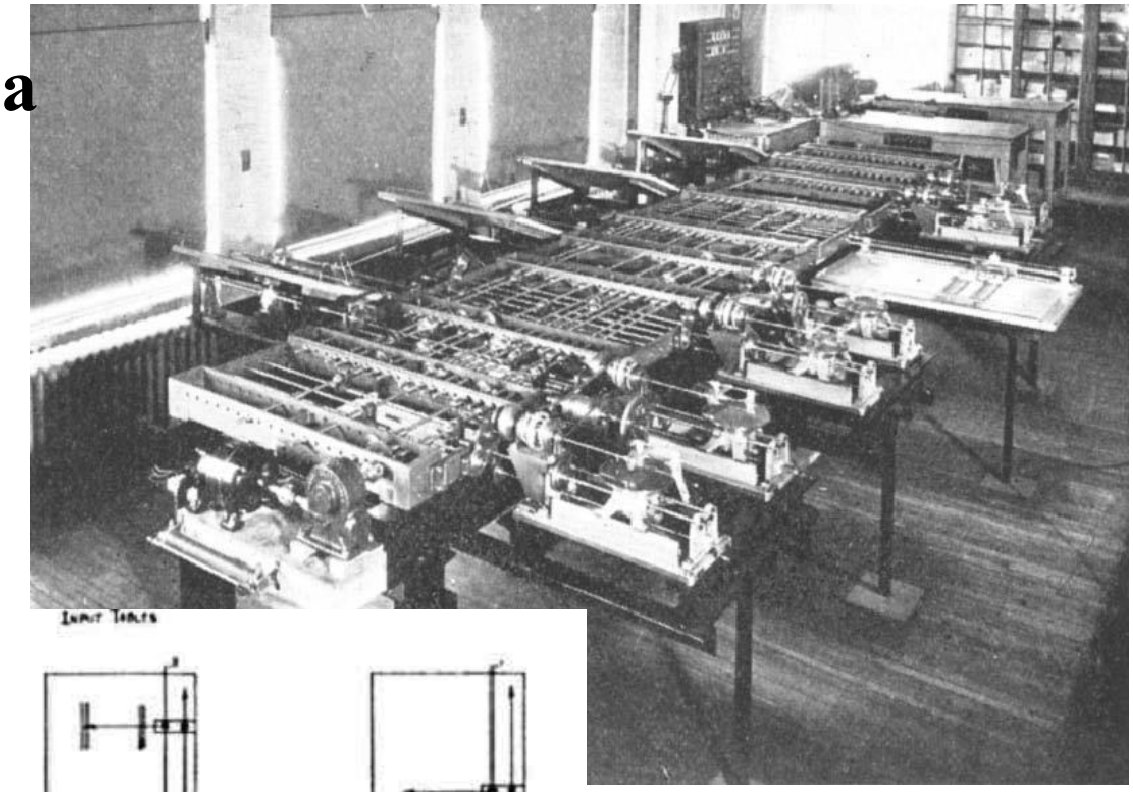


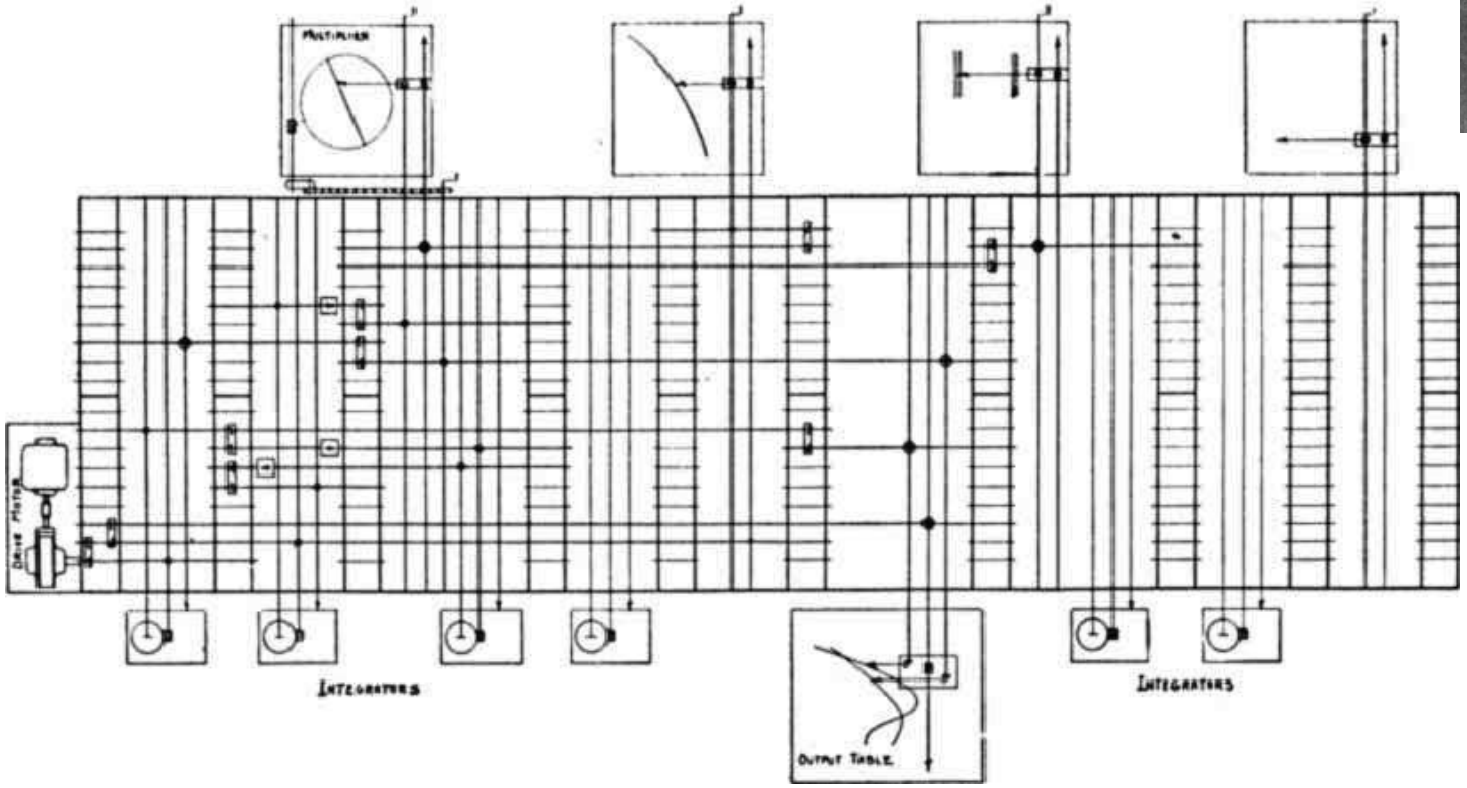


Sestavljanje računskega stroja

Diferencialni analizator

MIT (1931) [8]





Primer 1: harmonično nihanje

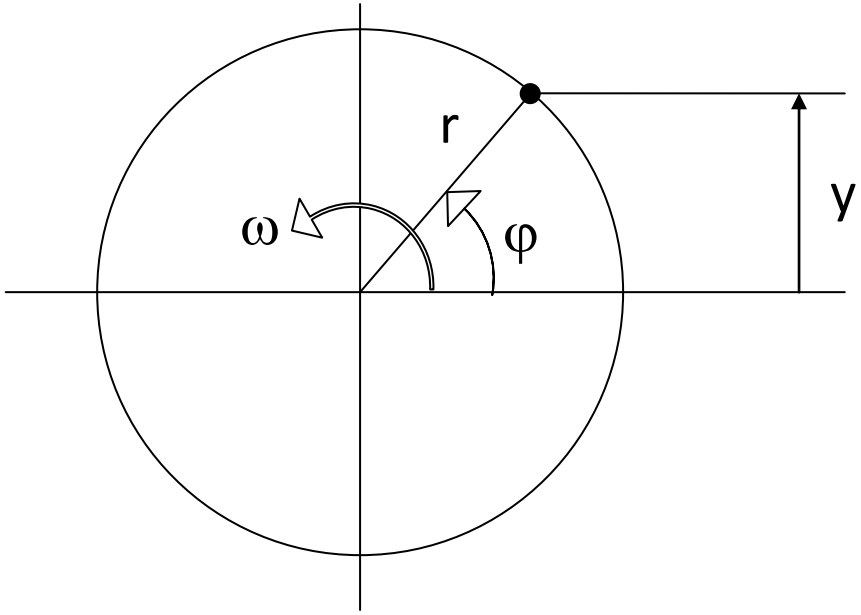
$$\frac{d\varphi}{dt} = \omega = konst.$$

$$y = r\ \sin\varphi = r\ \sin(\omega t)$$

$$\frac{dy}{dt} = -r\,\omega\ \cos(\omega t)$$

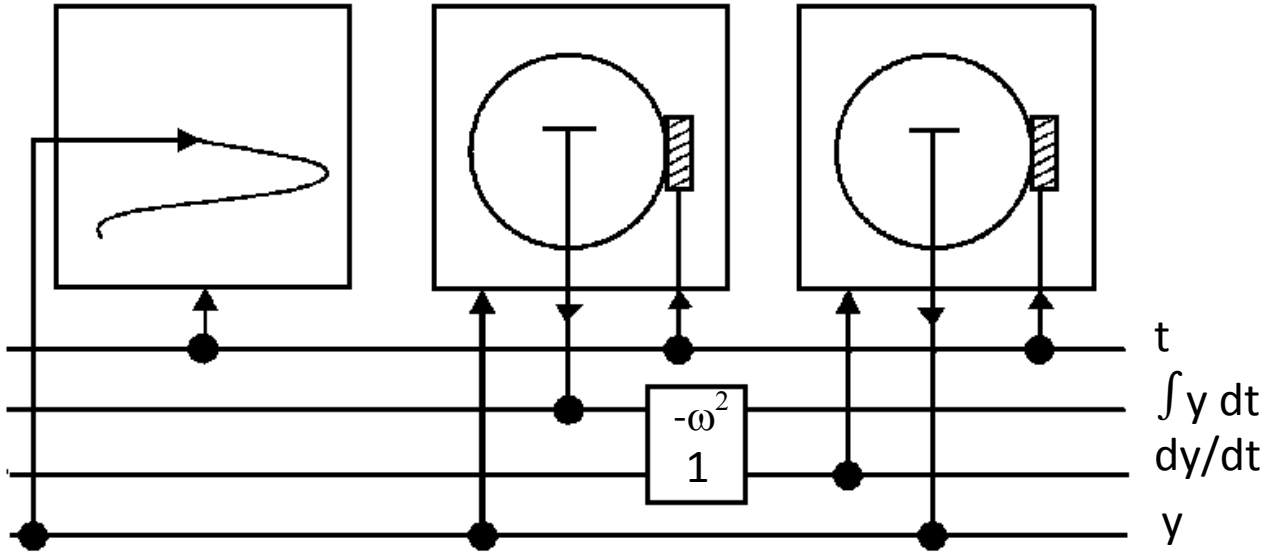
$$\frac{d^2y}{dt^2} = -r\,\omega^2 \sin(\omega t) = -\omega^2\, y$$

$$\boxed{\frac{d^2y}{dt^2} = -\omega^2 y}$$



Preuredimo:

$$\frac{dy}{dt} = -\omega^2 \int y\, dt$$

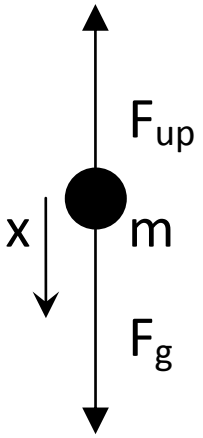


Vir: [9]

Primer 2: padajoča masna točka

a) Linearni zakon upora

$$m\vec{a} = \sum_{i=1}^{n} \vec{F}_i$$



os x: $m\,a_x = -b\,v_x + m\,g$

uredimo, upoštevamo K=b/m

$$\frac{d^2x}{dt^2} + K\frac{dx}{dt} - g = 0$$

Preuredimo:

$$\frac{dx}{dt} = -\int \left(K\frac{dx}{dt} - g\right) dt$$

Vir: [8]

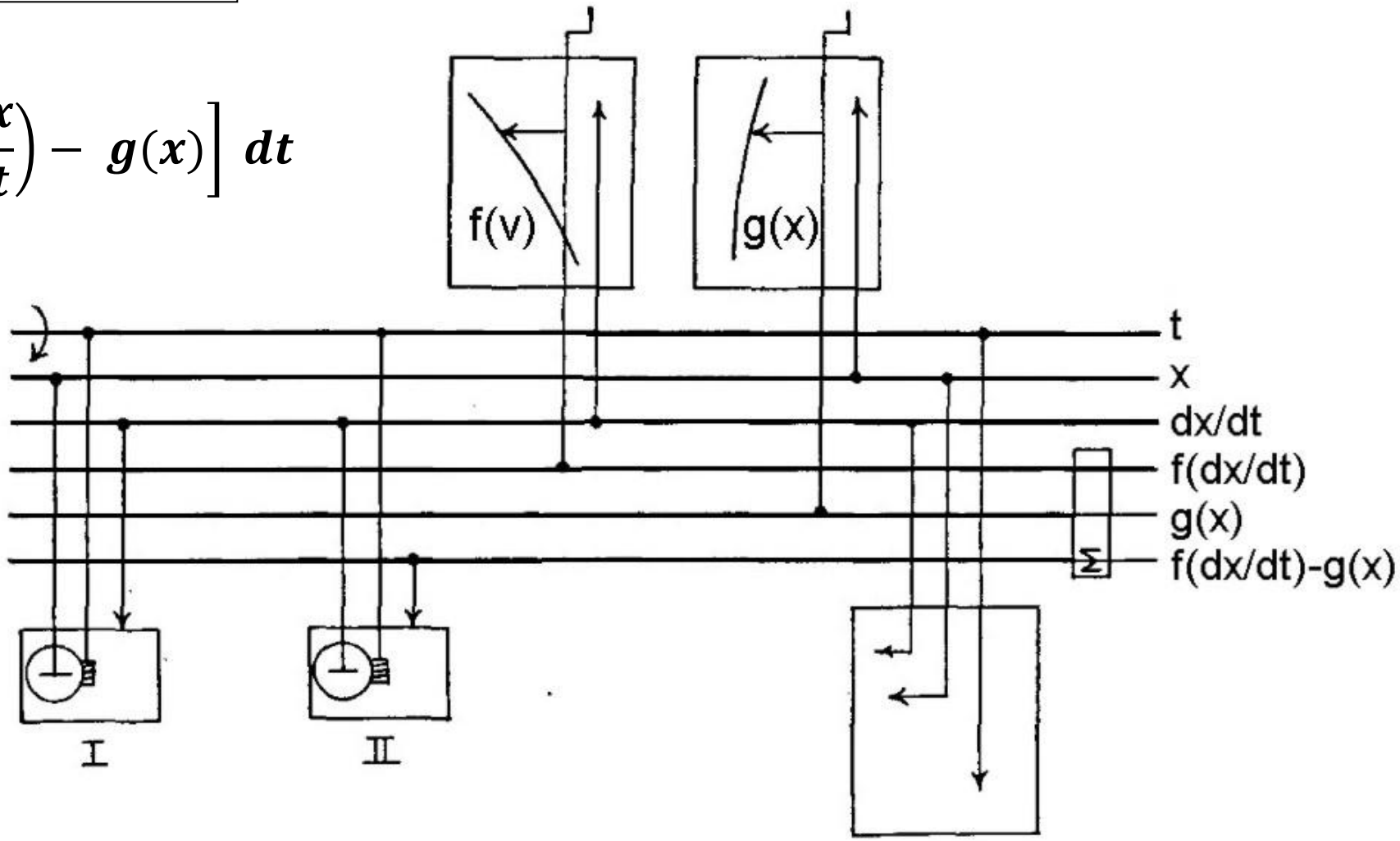
b) Nelinearni zakon upora:

$\vec{F}_{up} = m\, f(\vec{v})$, **dodatno še: g=g(x)**

$$\frac{d^2x}{dt^2} + f\left(\frac{dx}{dt}\right) - g(x) = 0$$

Preuredimo:

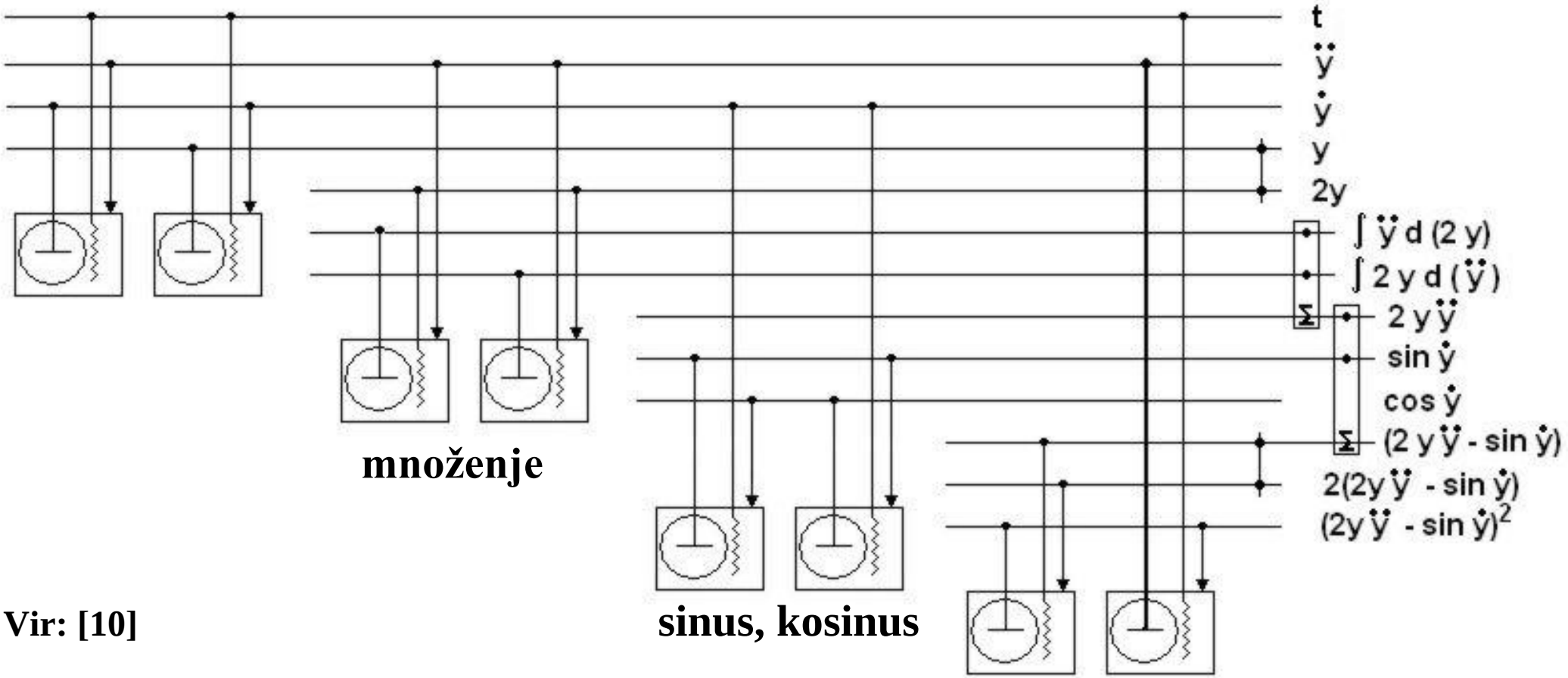
$$\frac{dx}{dt} = -\int \left[f\left(\frac{dx}{dt}\right) - g(x) \right] dt$$



Vir: [8]

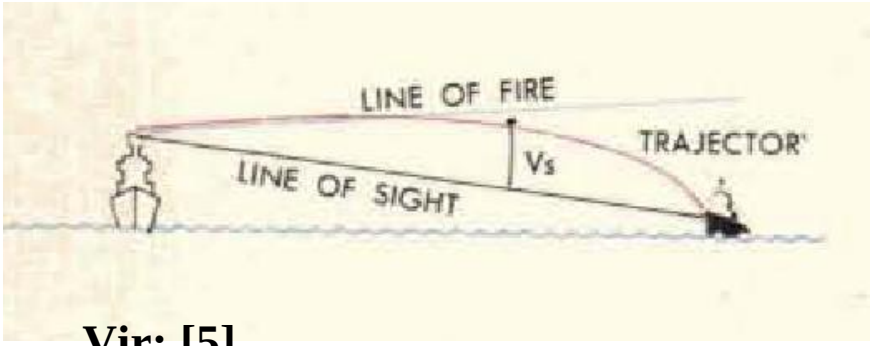
Primer 3

$$\frac{d^3y}{dt^3} = \left[2y\,\frac{d^2y}{dt^2} - \sin\left(\frac{dy}{dt}\right)\right]^2$$

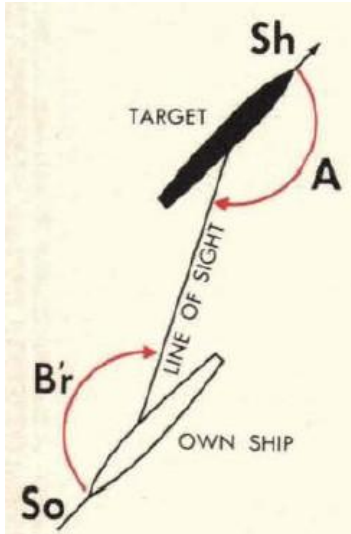


Vir: [10]

Primer 4



Vir: [5]



In order to find Sight Angle, *Vs*, these six quantities are used as inputs to the network:

1. Own Ship Speed, *So*, which is assumed to come in automatically from a pitometer log, by synchro transmission.
2. Director Train, *B'r*, which could come in automatically from a director, also by synchro transmission.
3. Target Speed, *Sh*, which might be estimated at the director and be phoned down to the Computer and set in by hand.
4. Target Angle, *A*, which might also be estimated at the director and be phoned to the Computer and set in by hand.
5. Initial Range setting, *jR*, which could be phoned down from the Director and set into the Computer by hand.
6. Time, *T*, which is put in automatically by the time motor.

$$Vs = f\{\{[-So\cos B'r - Sh\cos A] \times [K(\int dR \times \Delta T + jR) + c]\} + \{\int dR \times \Delta T + jR\}\}$$

Literatura:

[1] http://www.meccano.us/differential_analyzers/robinson_da/index.html, (sept. 2015).

[2] http://history-computer.com/, (sept. 2015).

[3] Fluidika, *Tehnička enciklopedija*, *5. svezak*, Leksikografski zavod Miroslav Krleža, Zagreb.

[4] Mabie H.H, Ocvirk F.W., *Mechanisms and Dynamics Machinery*, *Third Edition*, John Wiley and Sons, 1978.

[5] *Basic Fire Control Mechanisms*, *Ordnance Pamphlet 1140*, Department of the Navy, USA, 1944.

[6] Kasper J.E., *Construction and application of a mechanical differential analyzer*, *Dissertation*, State University of Iowa, 1955.

[7] https://en.wikipedia.org/wiki/Ball-and-disk_integrator, (sept. 2015).

[8] Bush V., *The Differential Analyzer. A New Machine for Solving Differential Equations*, Massachusetts Institute of Technology, 212, 1931.

[9] Robinson T., *The Meccano Set Computers, A history of differential analyzers made from children's toys*, IEEE Controls System Magazine, June 2006.

[10] Brooks C. et all, *The Marshall Differential Analyzer Project: A Visual Interpretation of Dynamic Equation*, Advances in Dynamical Systems and Applications, Volume 3 Number 1, 2008.

[11] http://www.meccano.us/differential_analyzers/robinson_da/vcf70.html - film prikazuje Meccano diferencialni analizavor v delovanju (sept. 2015).

[12] http://www.computerhistory.org/revolution/analog-computers/3/143/2394 film (sept. 2015).