
UČINEK METULJA
UROŠ KUZMAN
CHAOS THEORY


NETOPIRJA
UČINEK METULJA
UROŠ KUZMAN
CHAOS THEORY

SHERLOCK HOLMES

TAKOJ	PO 1h	JEZERO
9°C	7°C	5°C

Kdaj se je zgodil umor?

OHLAJANJE JE SORAZMERNO S TEMPERATURNO RAZLIKO!

$$T_n = T_{n-1} - k\left(T_{n-1} - T_j\right), k > 0$$

REKURZIVNO ZAPOREDJE

NAŠI PODATKI: $T_0 = 9^0\text{C}, T_1 = 7^0\text{C}, T_j = 5^0\text{C}$

IŠČEMO: $k > 0$ in $n < 0$, da bo $T_n = 37^0\text{C}$.

SHERLOCK HOLMES

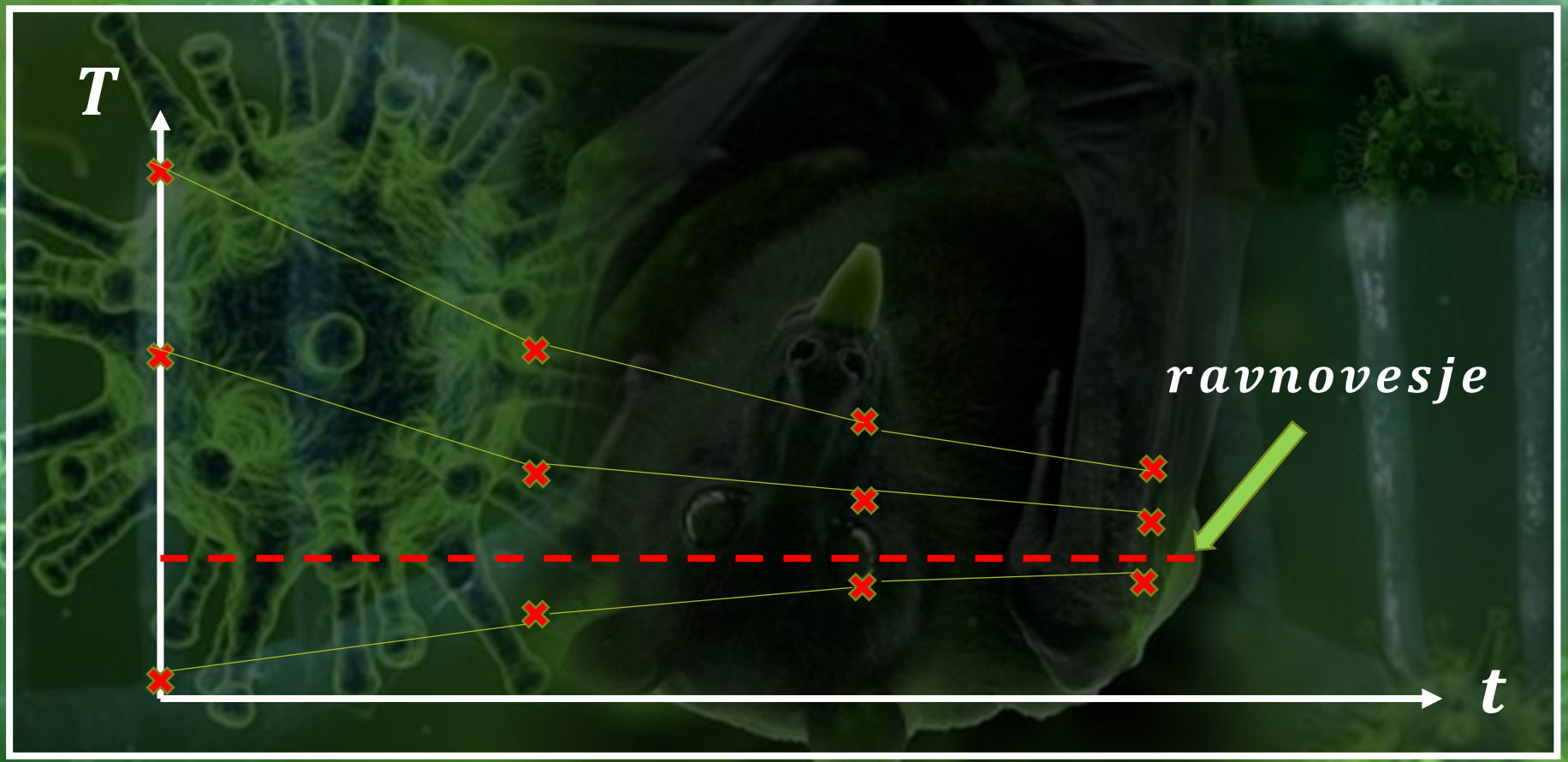
$$T_n = \frac{1}{2} T_{n-1} + \frac{5}{2}\,^0\mathrm{C}$$

FIKSNA OZ. RAVNOVESNA TOČKA $T_0 = 5\,^0\mathrm{C}$

EKSPLICITNA REŠITEV

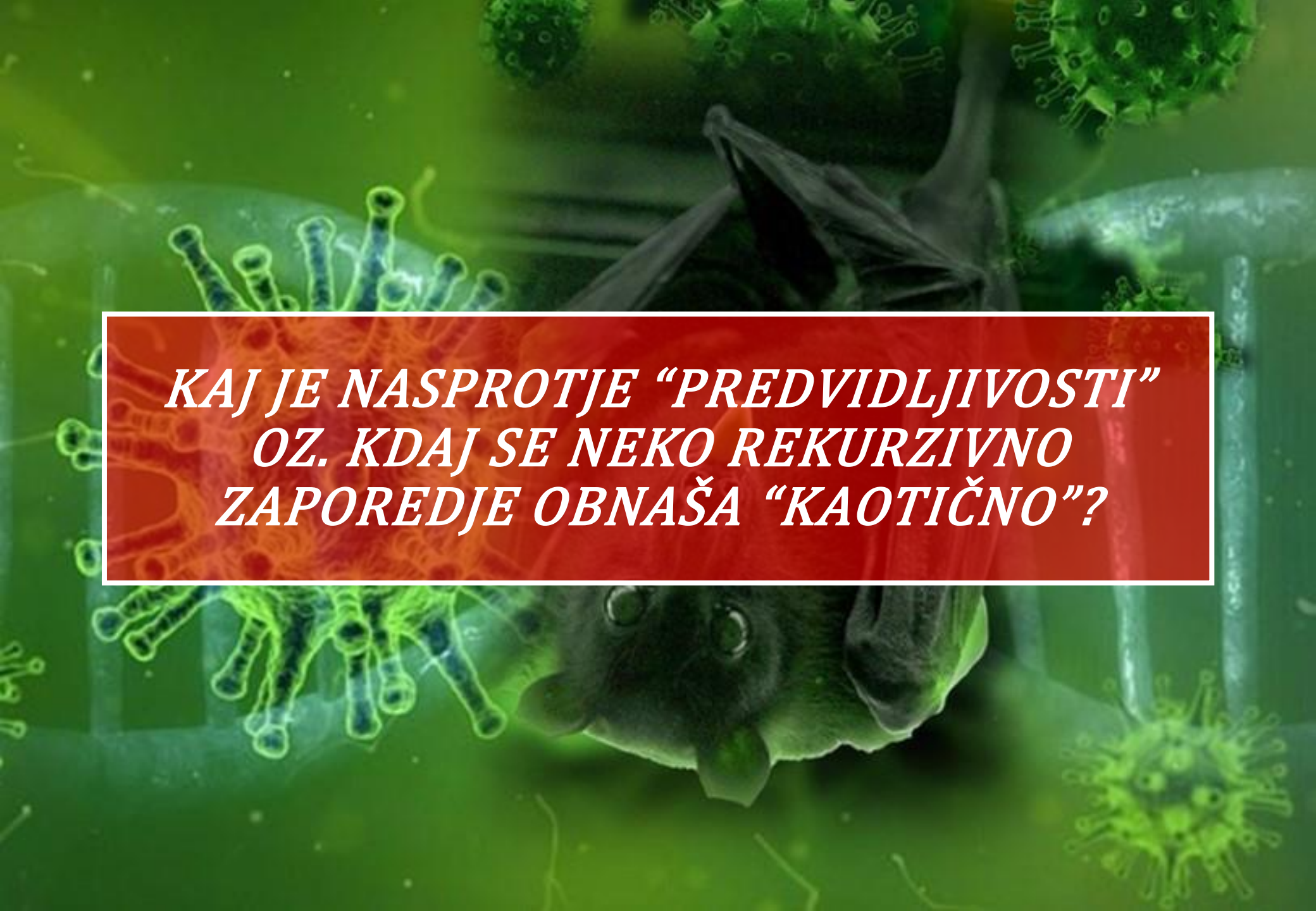
$$T_n = \left(\frac{1}{2}\right)^n (T_0 - 5\,^0\mathrm{C}) + 5\,^0\mathrm{C}$$

SHERLOCK HOLMES



SHERLOCK HOLMES

PREDVIDLJIVO


KAJ JE NASPROTJE “PREDVIDLJIVOSTI”
OZ. KDAJ SE NEKO REKURZIVNO
ZAPOREDJE OBNAŠA “KAOTIČNO”?

DEFINICIJA KAOSA

$$f: I \subseteq \mathbb{R} \to I$$
$$x_n = f(x_{n-1})$$

$f^{m+k}(x_0) = f^m(x_0)$ za neka $m \in \mathbb{N}_0$ in $k \in \mathbb{N}$.

TRIJE POGOJI:

- ***Gostost periodičnih točk***; Periodične točke funkcije f so goste na I.
- ***Tranzitivnost***; Za poljubna odprta podintervala $U_1 \in I$ in $U_2 \in I$ obstaja točka $x_0 \in U_1$ in $n > 0$ tako, da velja $f^n(x_0) \in U_2$.
- ***Občutljivostna konstanta***; Obstaja $\beta > 0$, da za vsak $x_0 \in I$ in vsak odprt interval U okrog točke x_0 obstajata točka $y_0 \in U$ in $n > 0$, da velja $|f^n(x_0) - f^n(y_0)| > \beta$.

TEJ LASTNOSTI PRAVIMO UČINEK METULJA

POENOSTAVLJENO

UČINEK METULJA POMENI, DA IMAMO POLJUBNO BLIZU ZAČETNE TOČKE TUDI TAKO, KI SE BO "OBNAŠALA POVSEM DRUGAČE" OZ. BO NJENO ZAPOREDJE "ŠLO DALEČ STRAN OD OSNOVNEGA"!

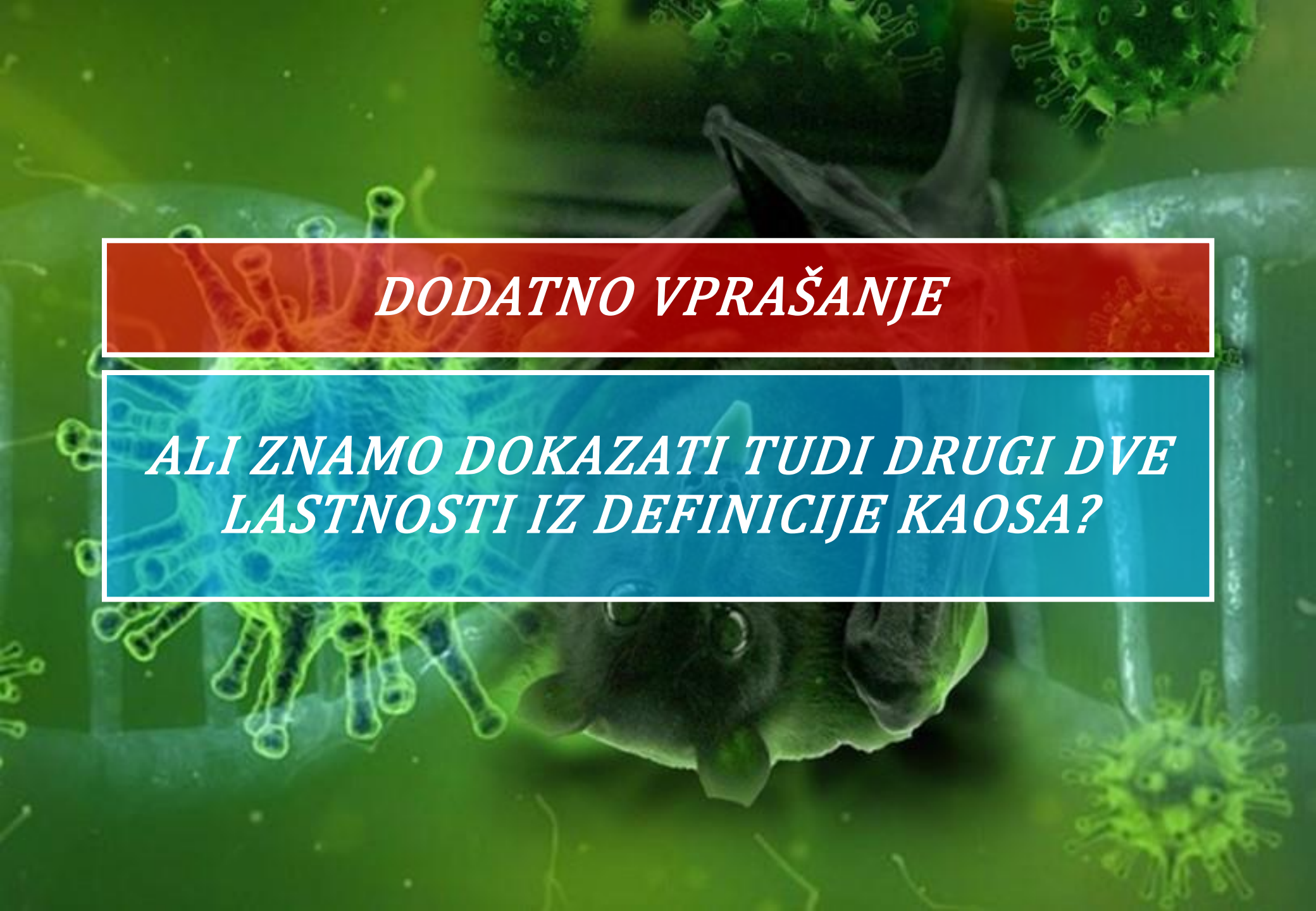
PRIMER: PODVOJITVENA PRESLIKAVA

$$f: [0, 1) \to [0, 1)$$
$$f(x) = 2x - \lfloor 2x \rfloor$$

ŠTEVILO PODVOJIMO IN MU ODŠTEJEMO 1, ČE JE TREBA.

DOKAŽI: "UČINEK METULJA"


NAMIG
BINARNI DECIMALNI ZAPIS!

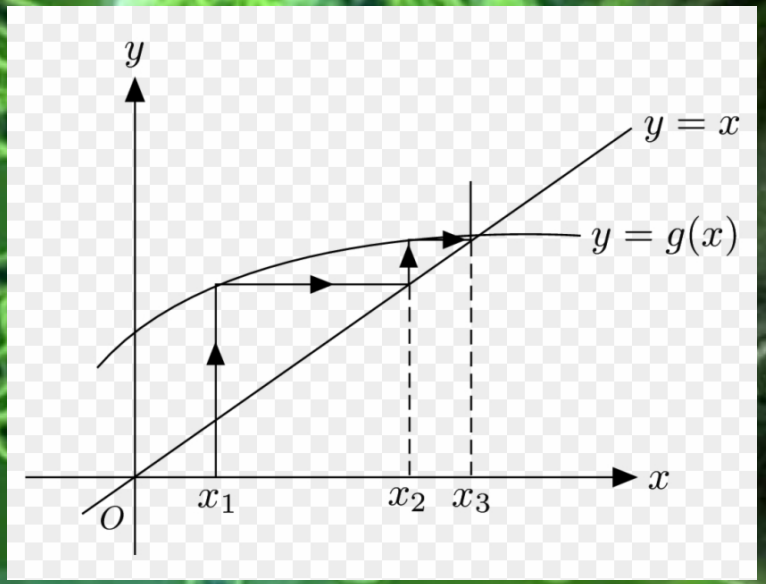


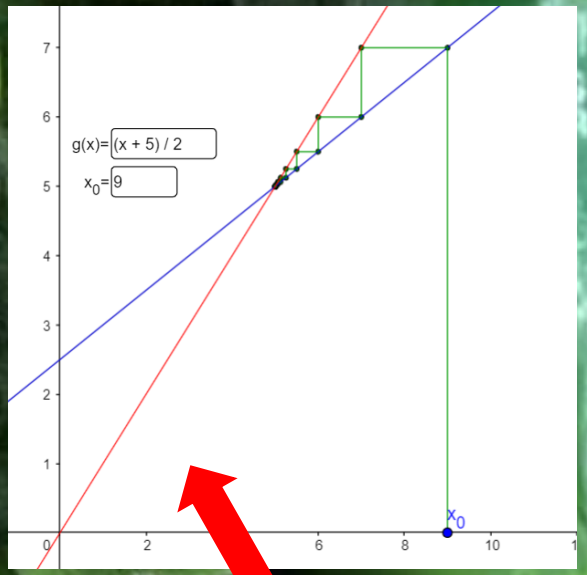
DODATNO VPRAŠANJE

ALI ZNAMO DOKAZATI TUDI DRUGI DVE LASTNOSTI IZ DEFINICIJE KAOSA?

PAJČEVINASTI DIAGRAM

PRI VIZUALIZACJI RAZVOJA ZAPOREDJA SI LAHKO POMAGAMO Z GRAFOM FUKNCIJE.



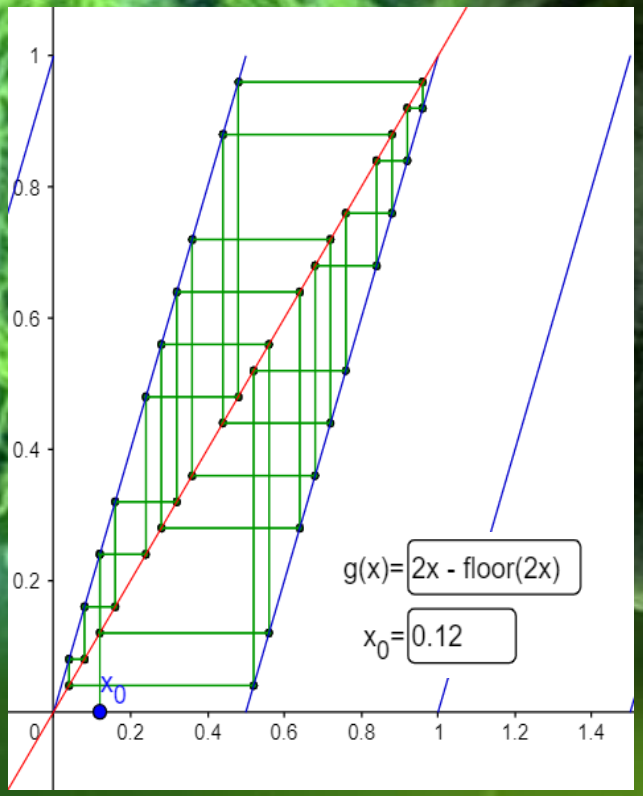


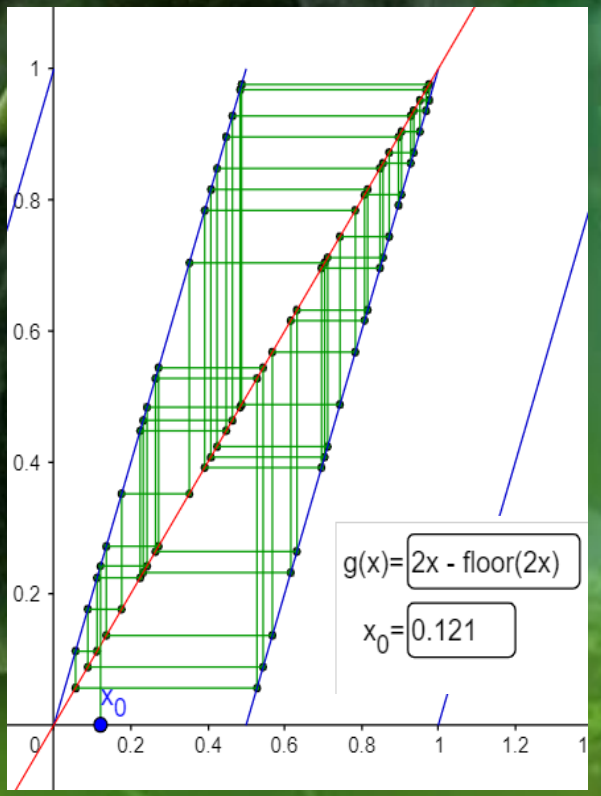
Cobweb plotter – GeoGebra

SHERLOCK

PRIMER: PODVOJITVENA PRESLIKAVA

$$f: [0, 1) \to [0, 1)$$
$$f(x) = 2x - \lfloor 2x \rfloor$$

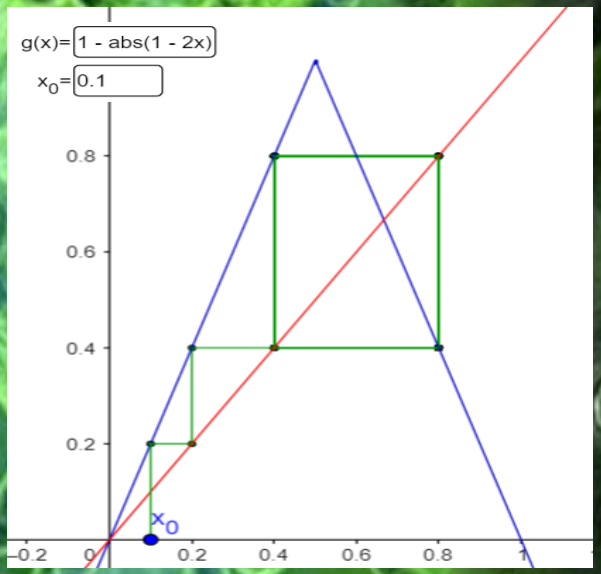


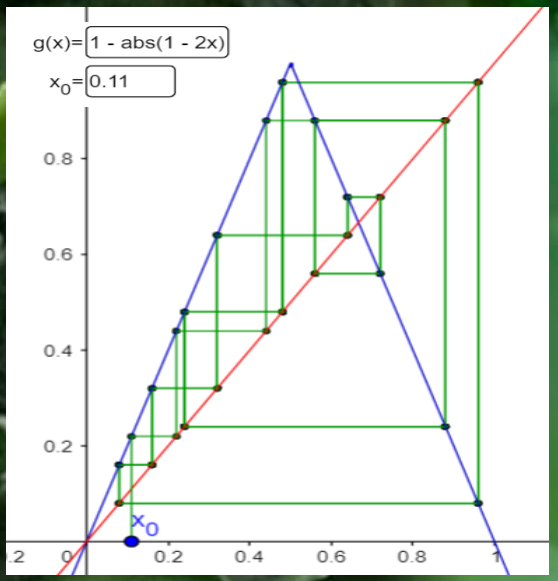



DOMAČA NALOGA

ŠOTORSKA PRESLIKAVA

$$f: [0,1) \to [0,1)$$
$$f(x) = 1 - |1 - 2x| = 2\ min(x, 1 - x)$$

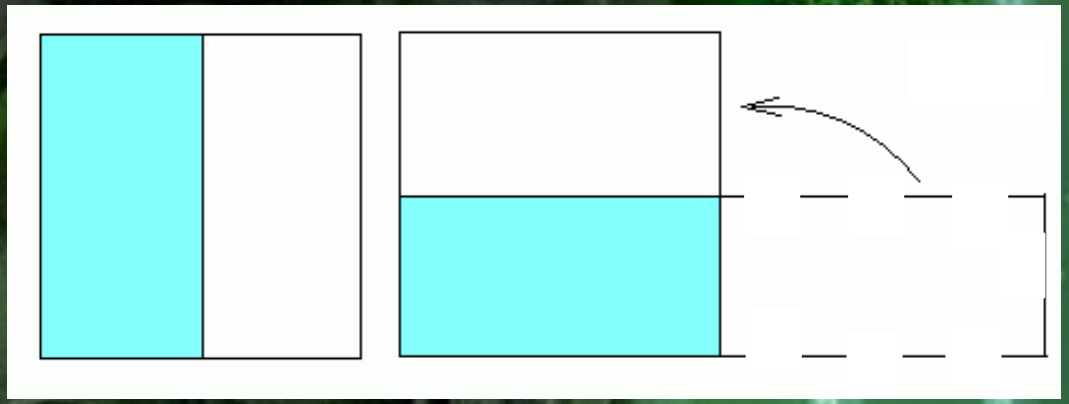




Z BINOMSKIM ZAPISOM DOKAŽI, DA JE TA PRESLIKAVA KAOTIČNA!


“UPORABEN” PRIMER

PRIMER: PEKOVA PRESLIKAVA

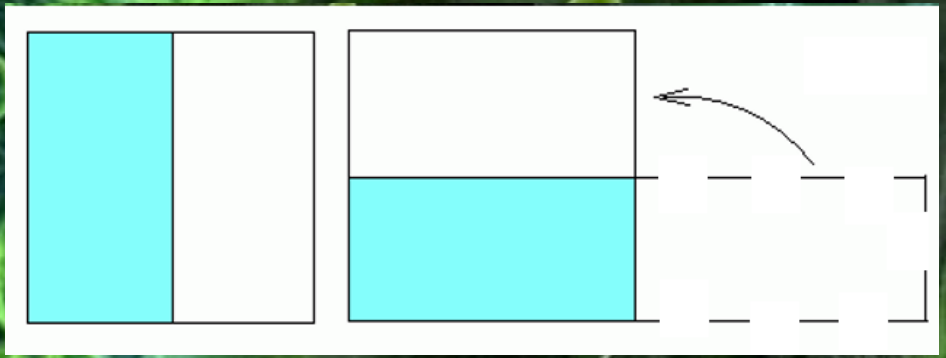


VIŠINO TESTA RAZPOLOVIMO, GA PREREŽEMO NA POL IN DESNO POLOVICO POLOŽOMO NA LEVO.

PRIMER: PEKOVA PRESLIKAVA

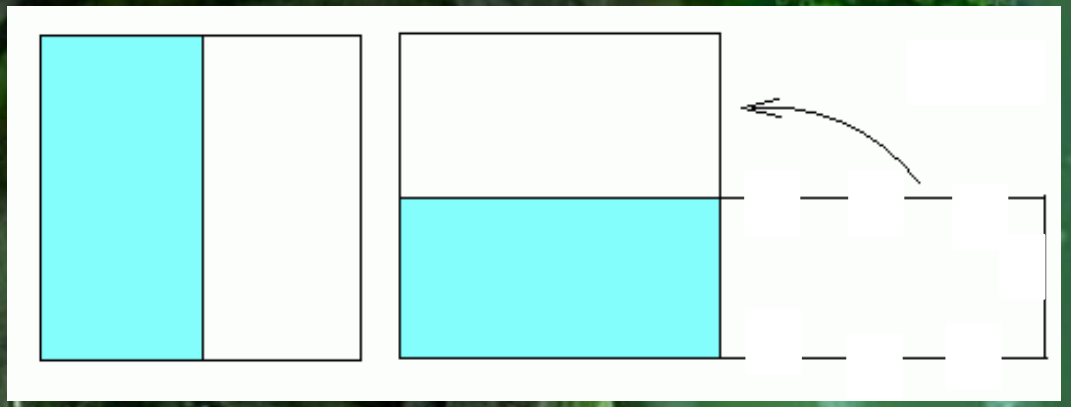
$$f: [0,1) \times [0,1) \to [0,1) \times [0,1)$$

$$f(x,y) = \left(2x - \lfloor 2x \rfloor, \frac{1}{2}(y + \lfloor 2x \rfloor)\right)$$



DOKAŽIMO "UČINEK METULJA"!

PRIMER: PEKOVA PRESLIKAVA



KAKO TA KAOS IZGLEDA V PRAKSI:

Baker's map mixing - Baker's map - Wikipedia

TERMINOLOŠKI SLOVAR

TERMINOLOŠKI SLOVAR

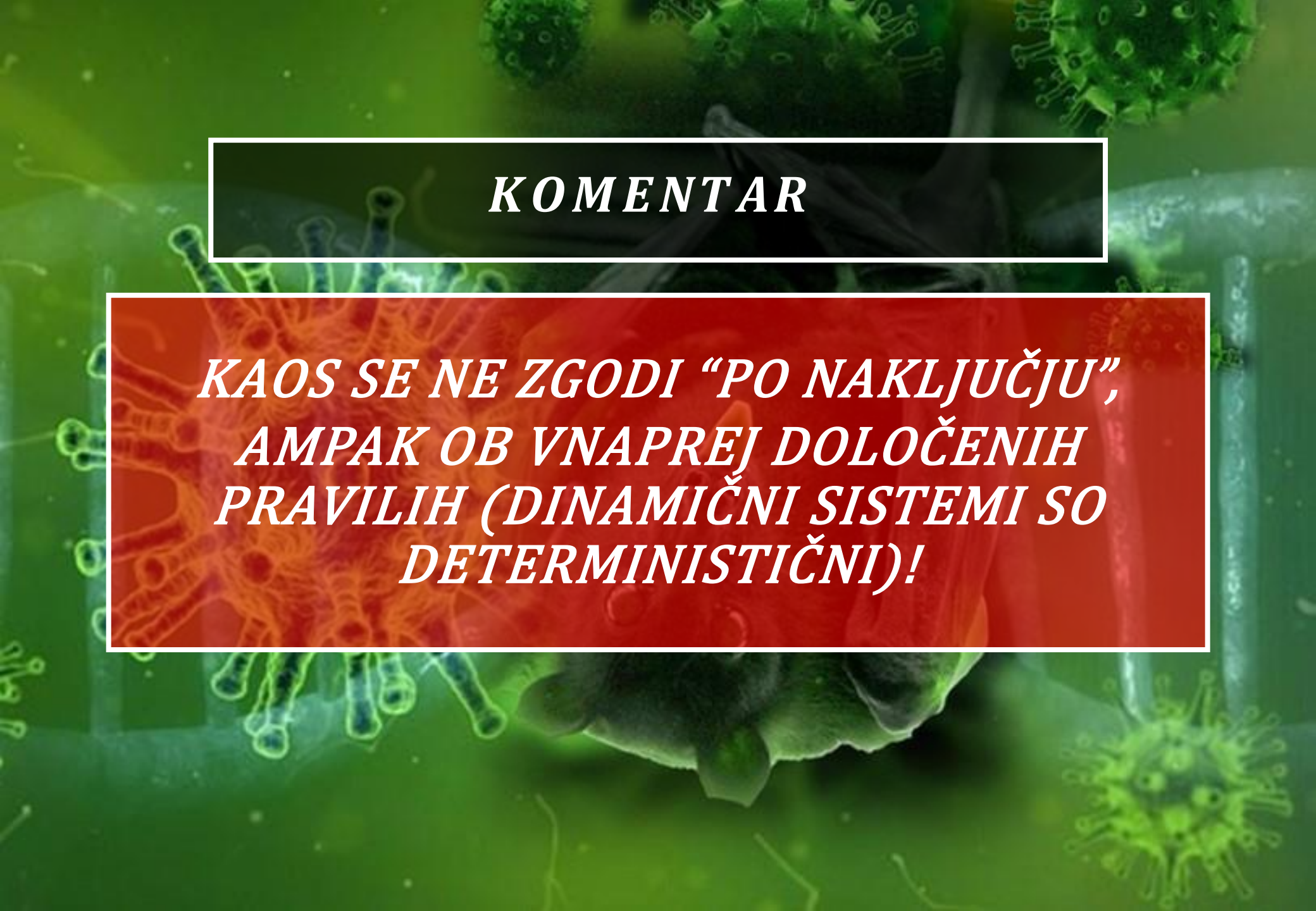
$$f : S \to S$$
$$x_n = f(x_{n-1})$$

n - naravno ali celo število

S - množica stanj, ki ni nujno v $\mathbb{R}^n$

TAKIM ZAPOREDJEM PRAVIMO TUDI <u>DISKRETNI DINAMIČNI SISTEMI</u>

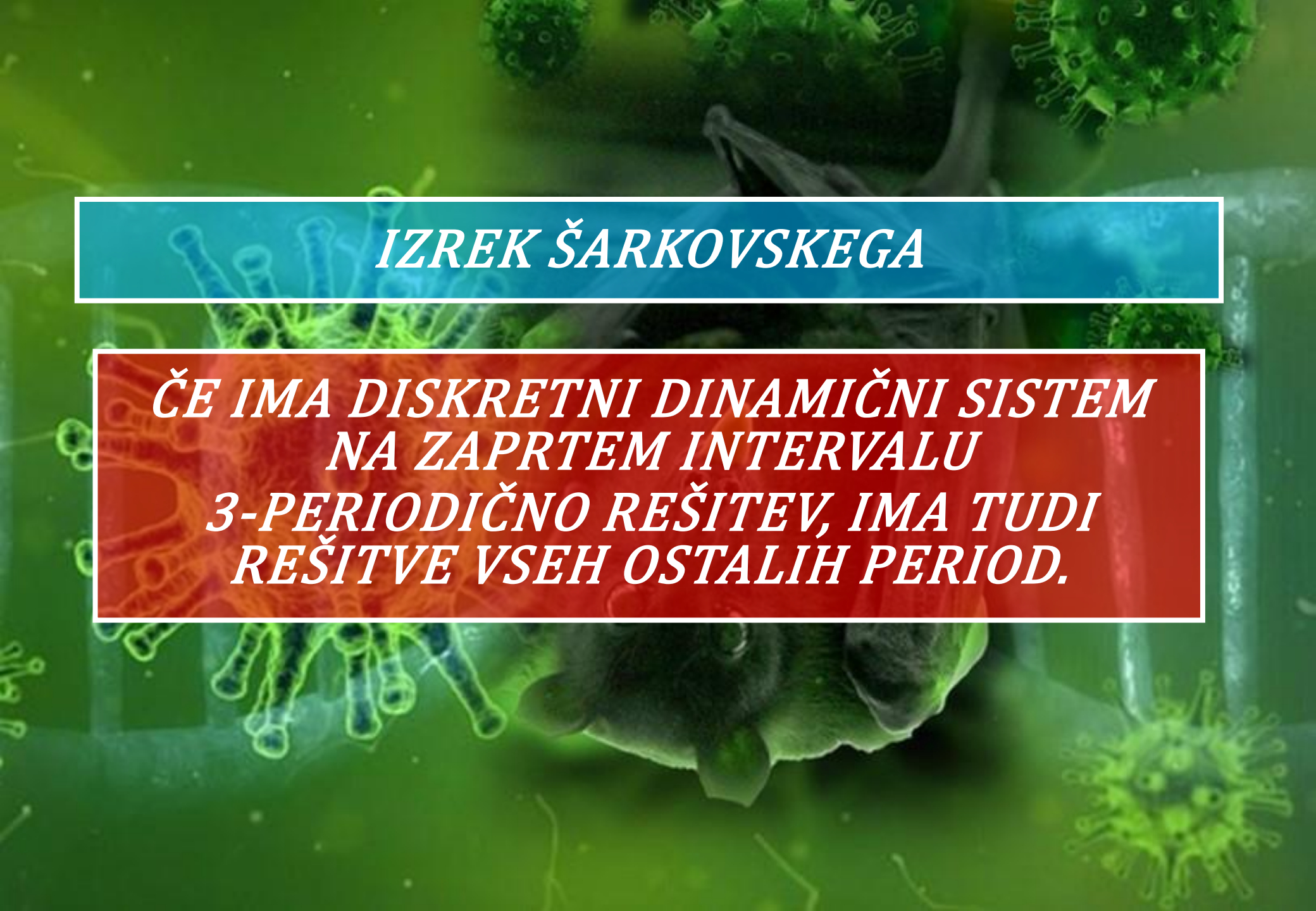
V SPLOŠNEM: DINAMIČNI SISTEM POMENI PRAVILO, KI POVE, KAKO SE "SPREMINJA" NEKO STANJE.



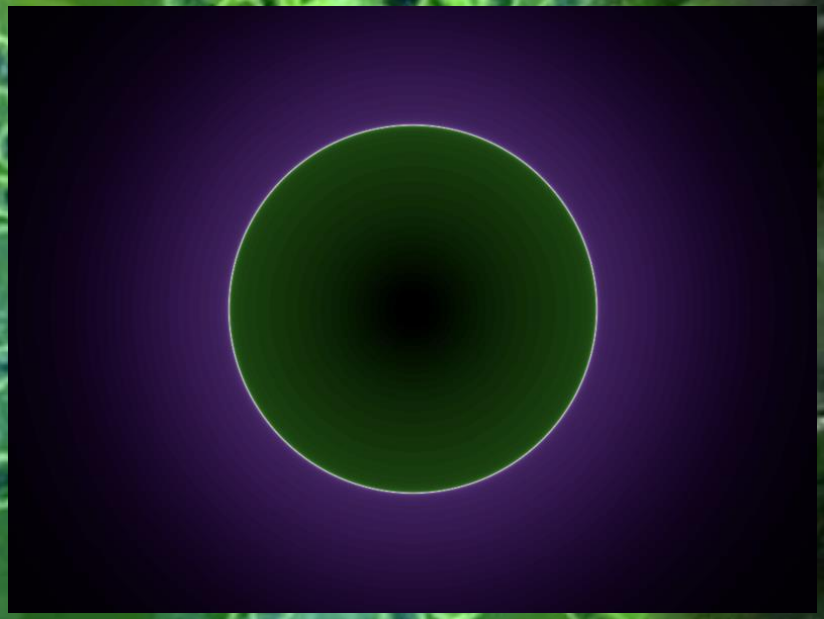
KOMENTAR

KAOS SE NE ZGODI “PO NAKLJUČJU”, AMPAK OB VNAPREJ DOLOČENIH PRAVILIH (DINAMIČNI SISTEMI SO DETERMINISTIČNI)!


ŠE NEKAJ ZANIMIVOSTI
IZ TEORIJE KAOSA!


IZREK ŠARKOVSKEGA
ČE IMA DISKRETNI DINAMIČNI SISTEM NA ZAPRTEM INTERVALU 3-PERIODIČNO REŠITEV, IMA TUDI REŠITVE VSEH OSTALIH PERIOD.

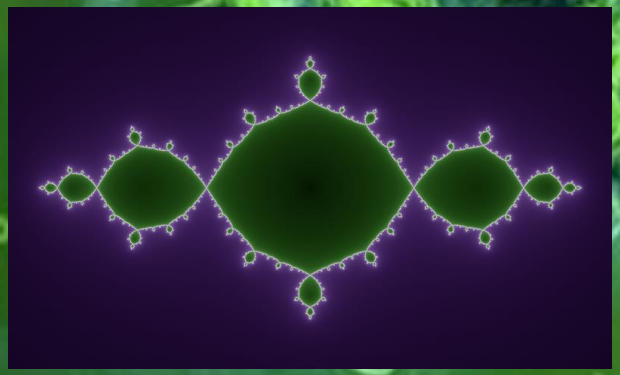
KOMPLEKSNA DINAMIKA

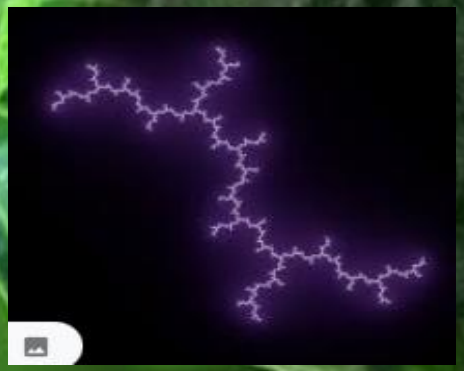


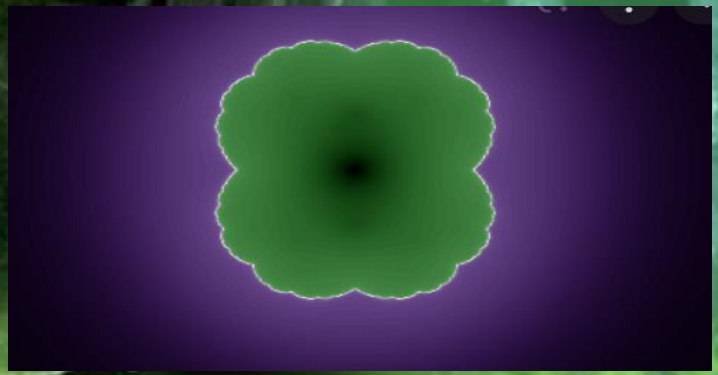
$$f: \mathbb{C} \to \mathbb{C}$$
$$f(z) = z^2$$

KOMPLEKSNA DINAMIKA

$$f: \mathbb{C} \to \mathbb{C}$$
$$f(z) = z^2 + c$$







PROBLEM IZ VIDIKA FIZIKA ...

ČAS NI DISKRETNA SPREMENLJIVKA!

REŠITEV

OD DISKRETNIH K ZVEZNIM SISTEMOM

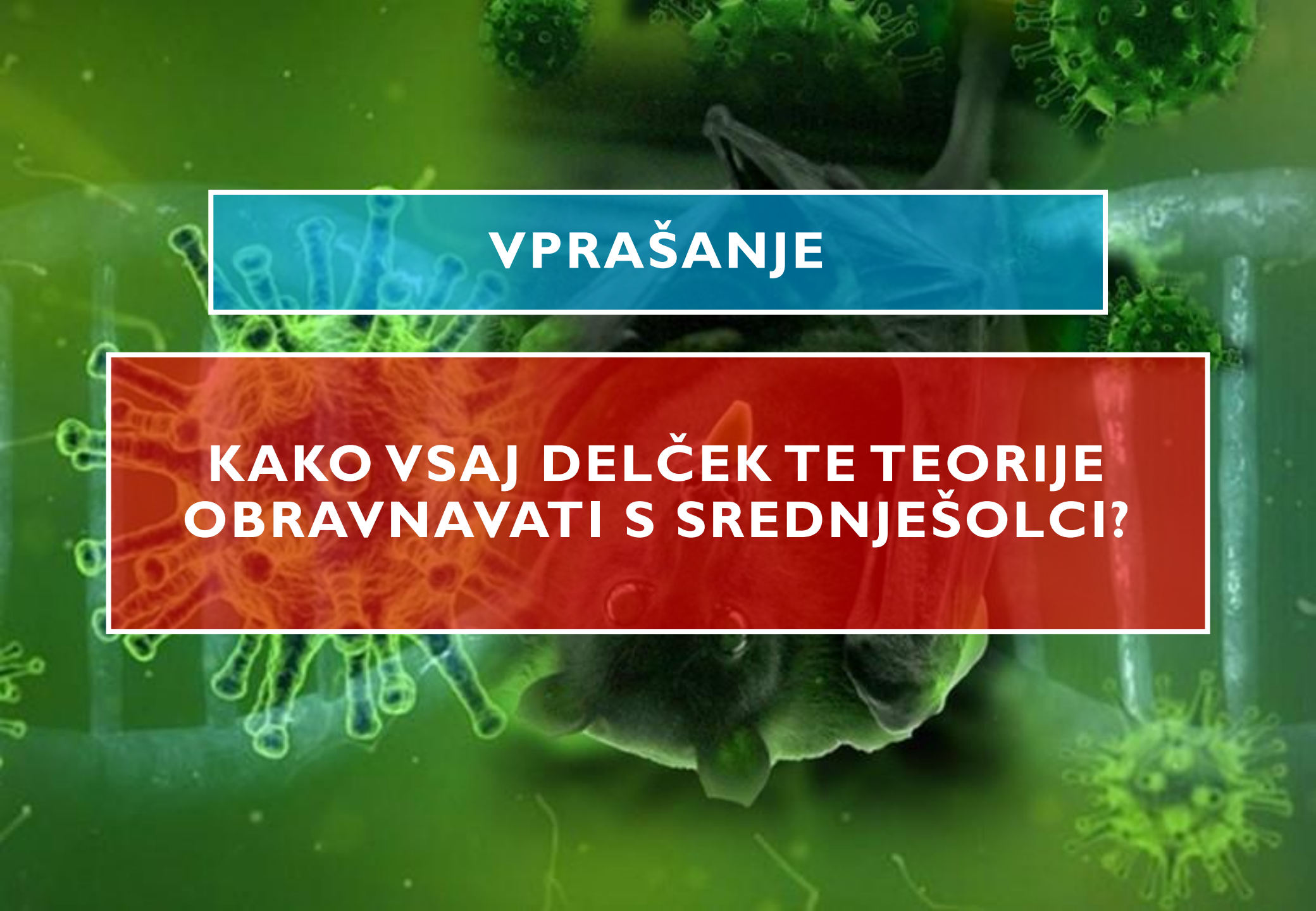
$$f : S \to S$$
$$x_n = f(x_{n-1})$$

n naravno ali celo število

$$f : S \to S$$
$$\dot{x} = f(x)$$

t realno število oz. čas

ZVEZNI DINAMIČNI SISTEM JE PODAN Z AVTONOMNO DIFERENCIALNO ENAČBO!


VPRAŠANJE
KAKO VSAJ DELČEK TE TEORIJE OBRAVNAVATI S SREDNJEŠOLCI?

ZVEZNI DINAMIČNI SISTEMI

Kako skakalcu izračunajo hitrost na odskočni mizi?

ZVEZNI DINAMIČNI SISTEMI

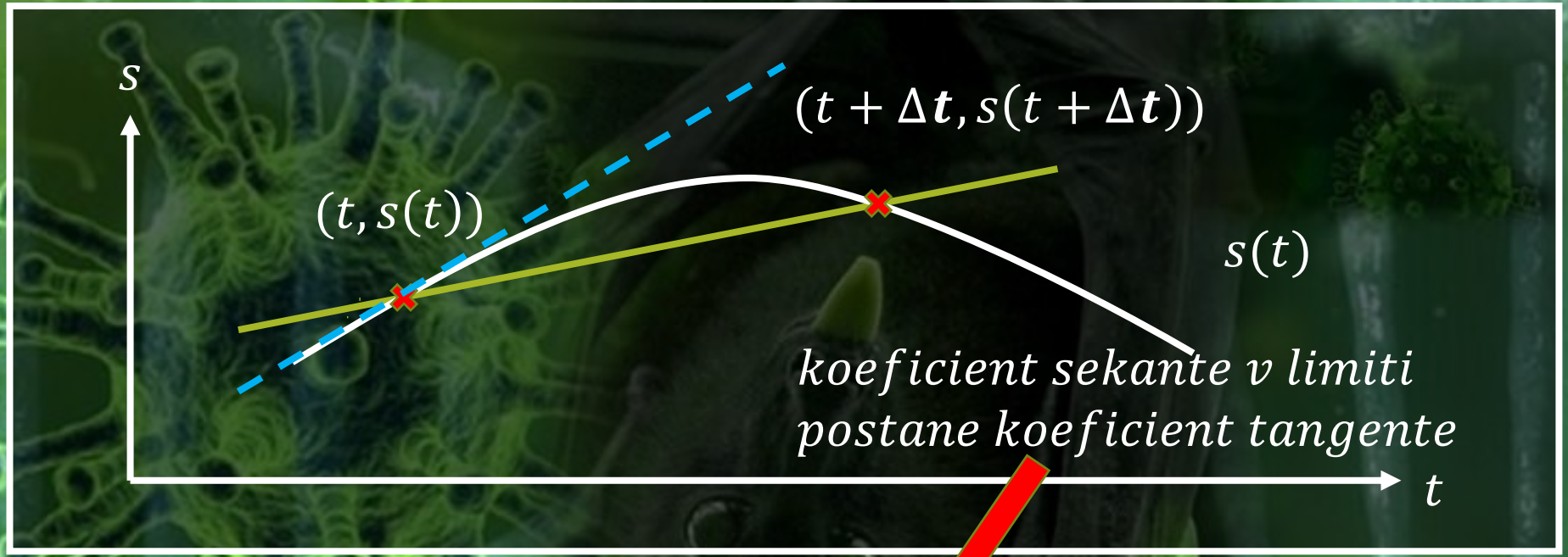
$$v = \frac{\Delta s}{\Delta t} \text{ za zelo majhen } \Delta t$$

ZVEZNI DINAMIČNI SISTEMI

$$v(t) = \lim_{\Delta t \to 0} \frac{s(t + \Delta t) - s(t)}{\Delta t} = \dot{s}(t)$$

odvod

GEOMETRIJSKI POMEN ODVODA



$$v(t) = \lim_{\Delta t \to 0} \frac{s(t + \Delta t) - s(t)}{\Delta t} = \dot{s}(t)$$

"ZVEZNI" SHERLOCK HOLMES

$$T_n = T_{n-1} - \frac{1}{2}\left(T_{n-1} - T_j\right)$$

HITROST SPREMINJANJA TEMPERATURE JE SORAZMERNA TEMPERATURNI RAZLIKI

$$\dot{T}(t) = -\frac{1}{2}\left(T(t) - T_j\right)$$

ALI ZNAMO TAKO ENAČBO REŠITI?

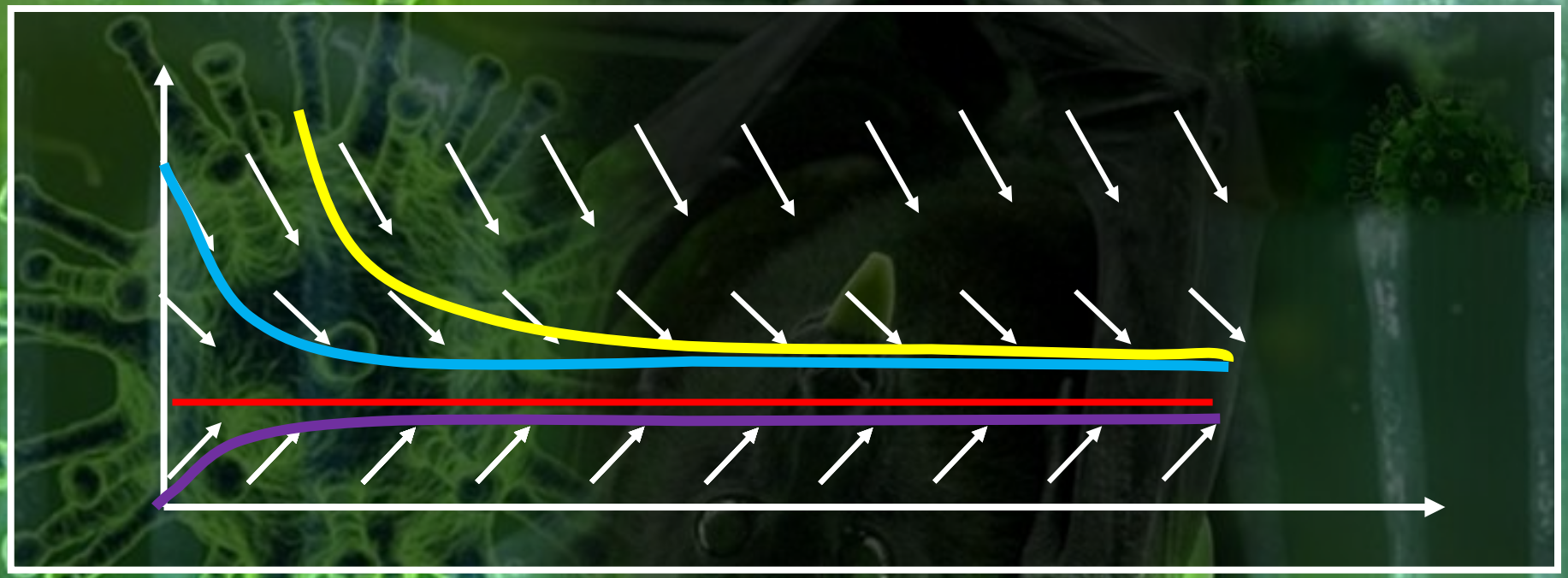
$$\dot{T}(t) = -\frac{1}{2}\left(T(t) - T_j\right)$$

ODGOVOR: EKSPLICITNO ŠELE NA FMF!
IMAMO PA NA VOLJO “GRAFIČNO METODO”.

ZVEZNI SHERLOCK HOLMES

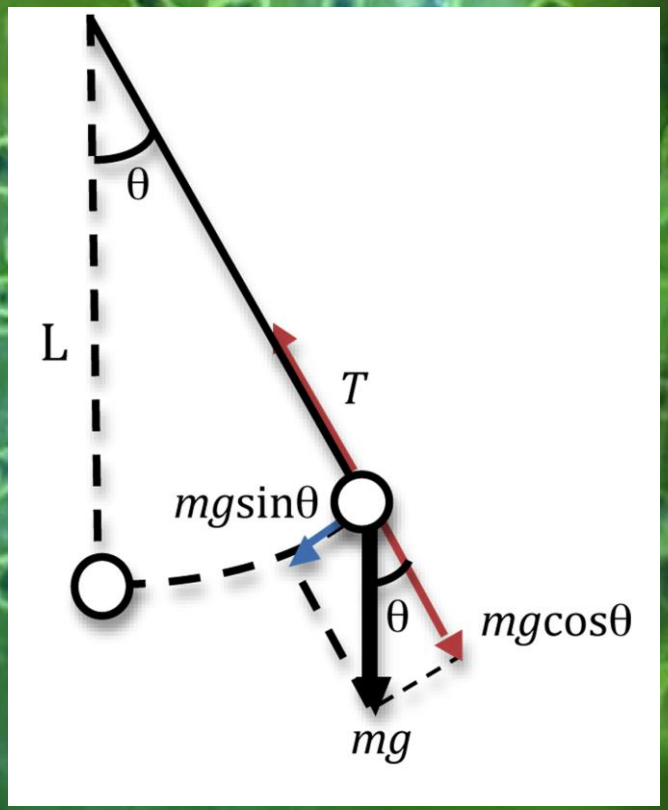
$$\dot{T}(t) = -\frac{1}{2}\left(T(t) - T_j\right)$$

ZVEZNI SHERLOCK HOLMES



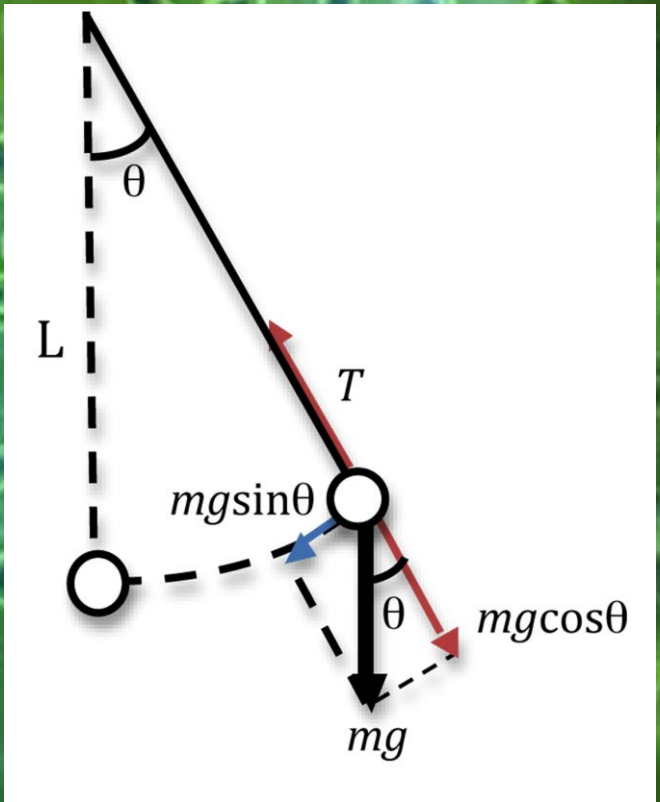
EKSPLICITNO: $T(t) = \left(\frac{1}{2}\right)^t (T_0 - T_j) + T_j$

PRIMER: NIHALO



$$F = ma = -mgsin(\theta)$$

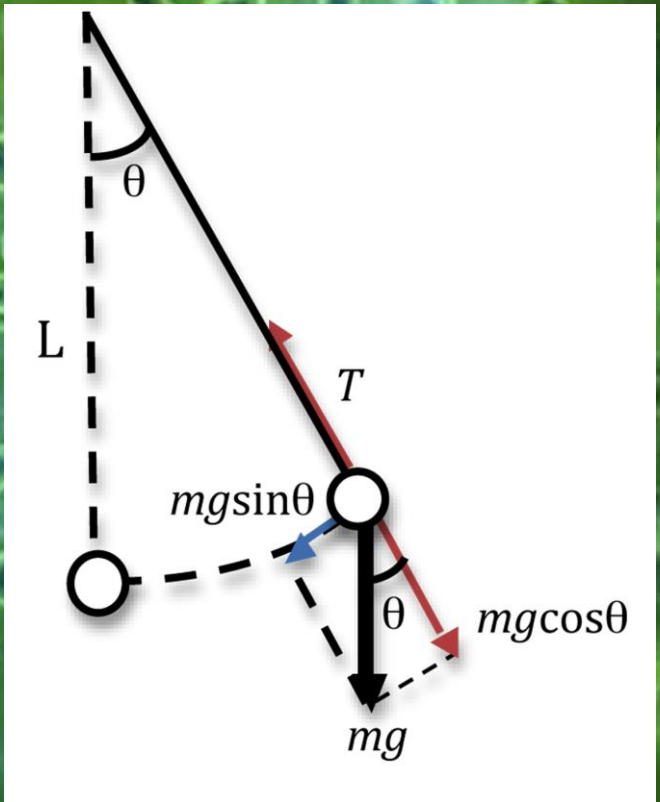
PRIMER: NIHALO



$$F = ma = -mgsin(\theta)$$

POSPEŠEK POMENI, KAKO HITRO SE SPREMINJA HITROST.

PRIMER: NIHALO

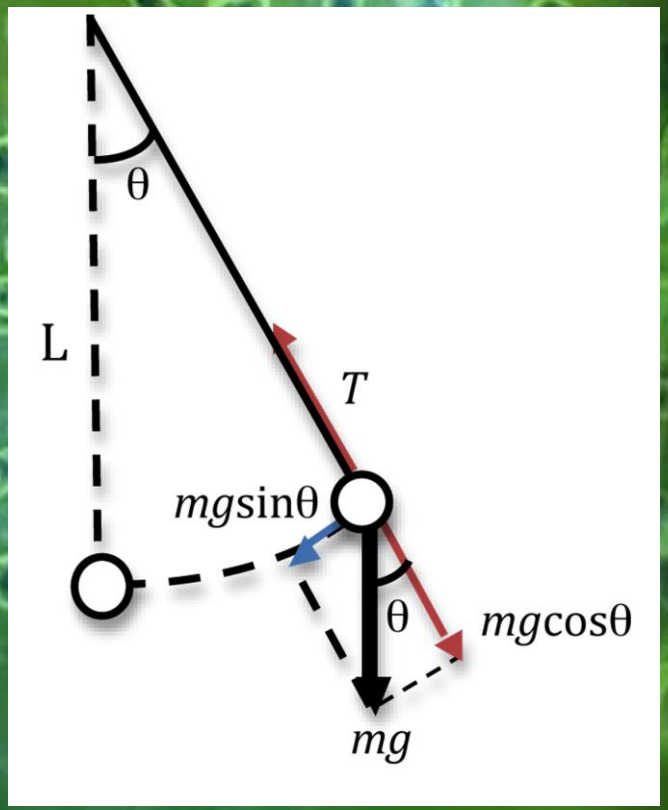


$$F = ma = -mgsin(\theta)$$

POSPEŠEK POMENI, KAKO HITRO SE SPREMINJA HITROST.

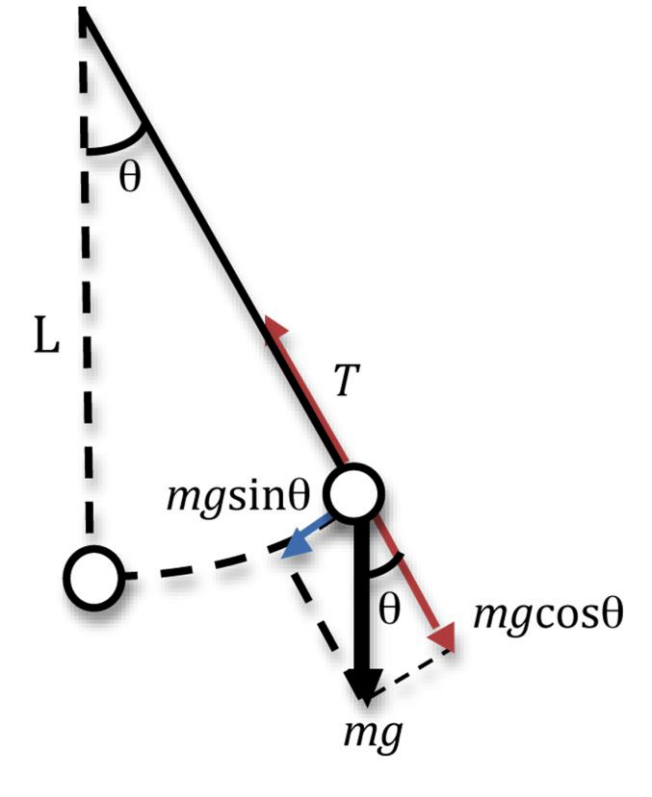
$$a = \dot{\omega}$$

PRIMER: NIHALO



$$F = \cancel{m}\ddot{\omega} = -\cancel{m}gsin(\theta)$$

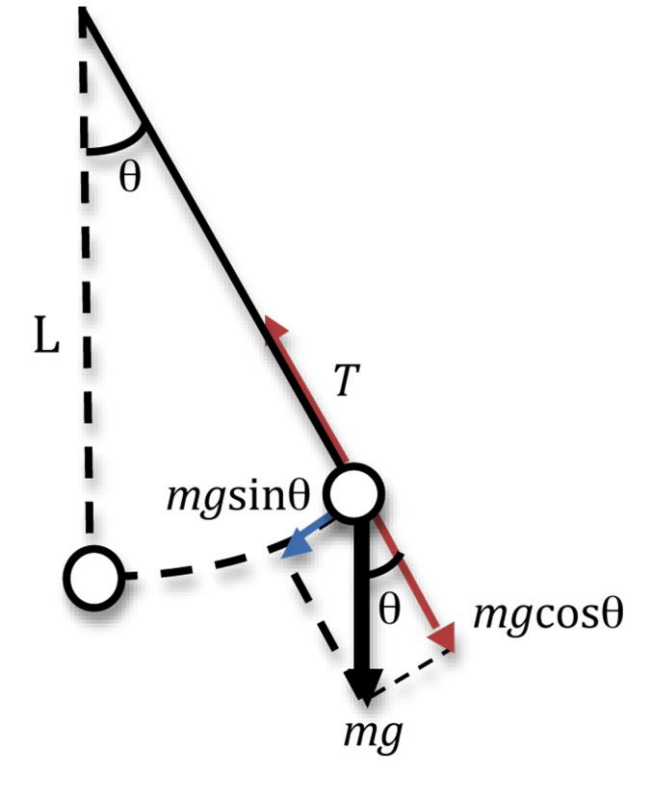
PRIMER: NIHALO



$$F = m\dot{\omega} = -mgsin(\theta)$$

DODAMO ŠE ENAČBO $\dot{\theta} = \omega$.

PRIMER: NIHALO



$$F = m\dot{\omega} = -mgsin(\theta)$$

DODAMO ŠE ENAČBO $\dot{\theta} = \omega$.

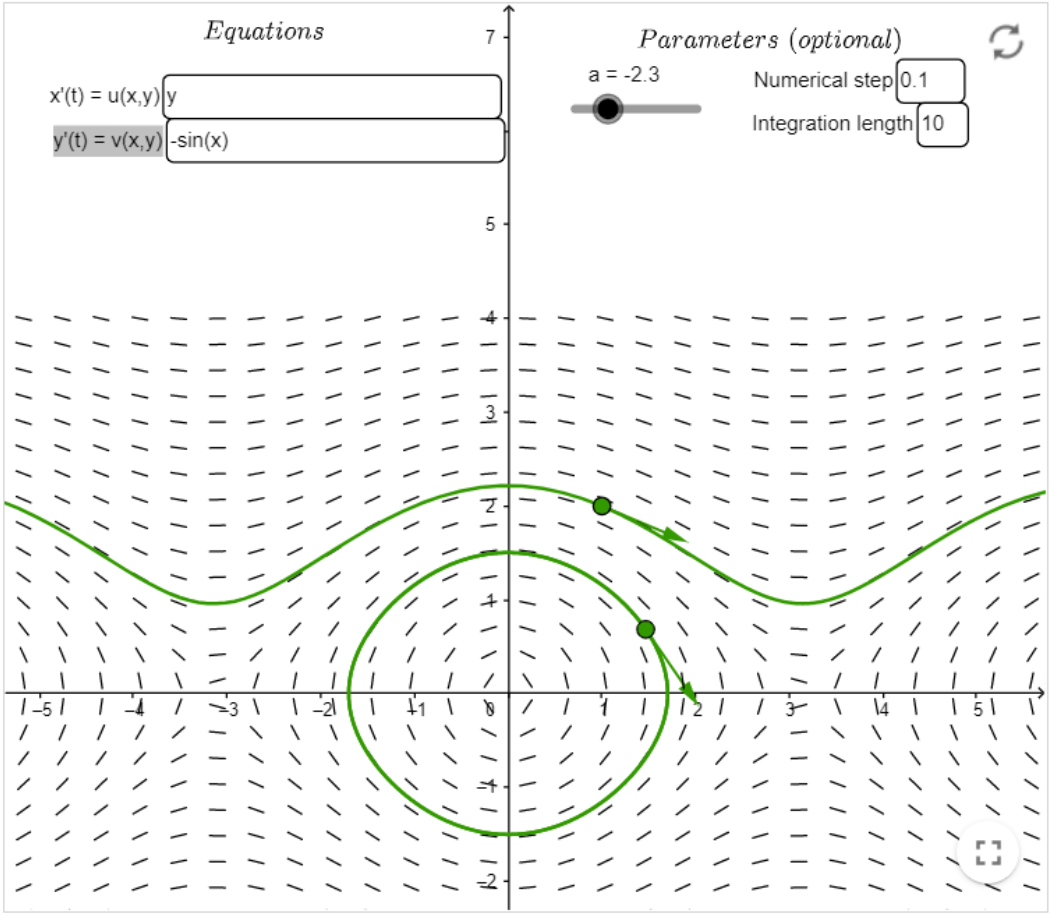
$$\dot{\theta} = \omega$$
$$\dot{\omega} = -gsin(\theta)$$

GEOMETRIJSKI POMEN

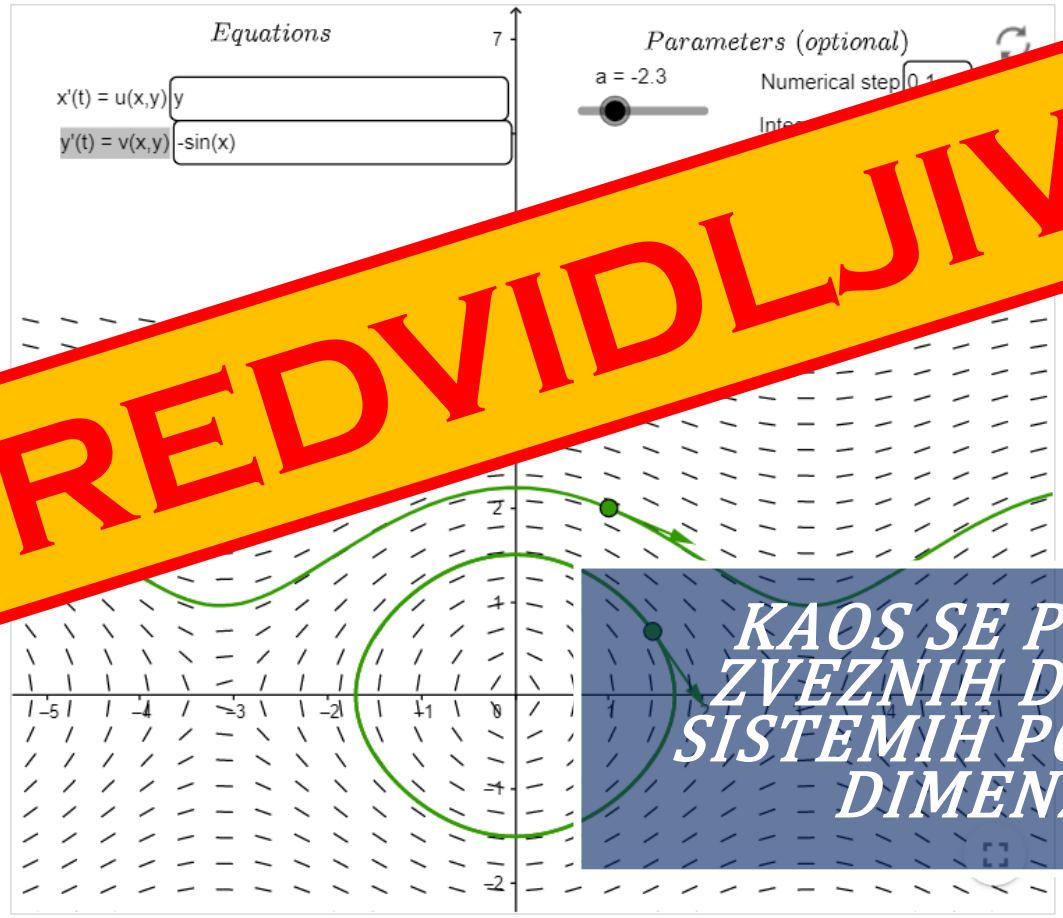
$(\dot{x}, \dot{y})$ pomeni tangentno smer

PRIMER: NIHALO

Phase plane – GeoGebra



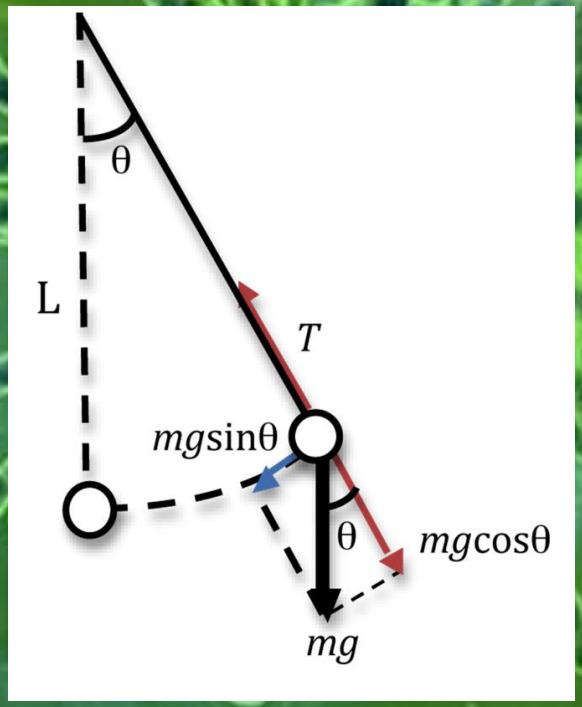
PRIMER: NIHALO



PREDVIDLJIVO

KAOS SE PRI "LEPIH" ZVEZNIH DINAMIČNIH SISTEMIH POJAVI ŠELE V DIMENZIJI ≥3!

REŠITEV: PODVOJIMO DIMENZIJO



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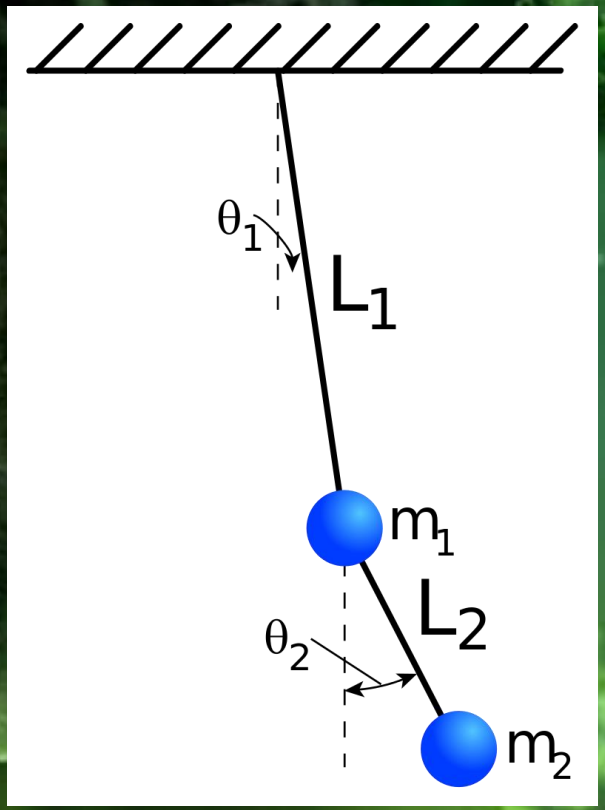


PRIMER: DVOJNO NIHALO

$$\theta_1' = \omega_1$$

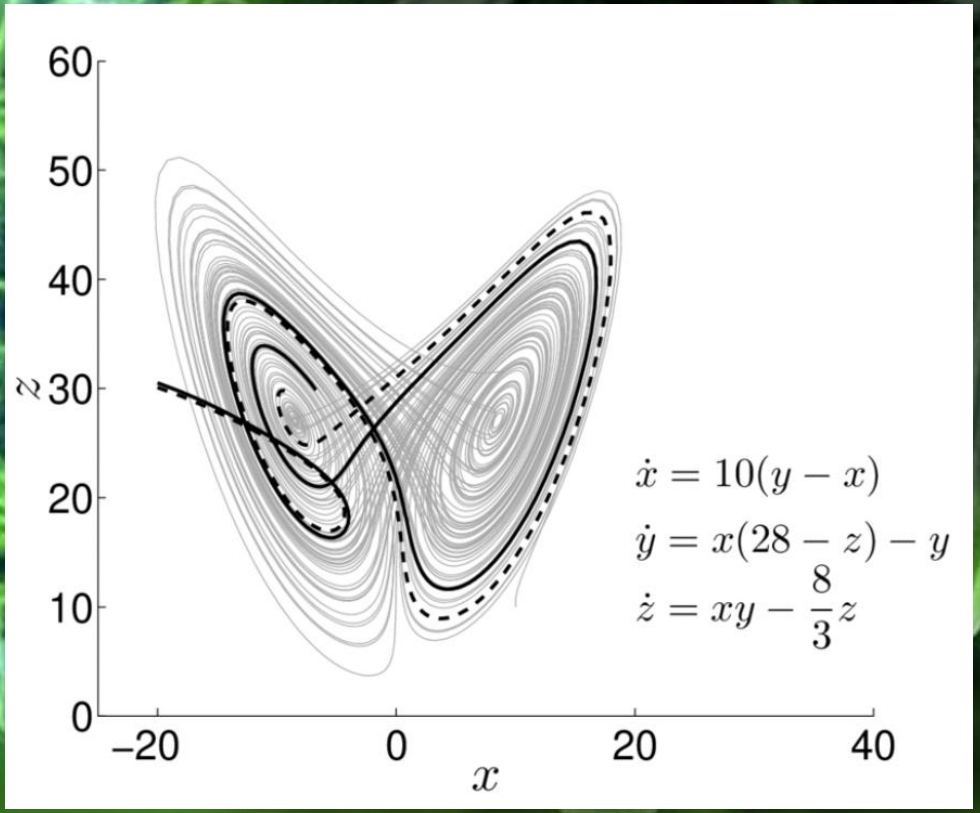
$$\theta_2' = \omega_2$$

$$\omega_1' = \frac{-g\,(2\,m_1 + m_2)\sin\theta_1 - m_2\,g\sin(\theta_1 - 2\,\theta_2) - 2\sin(\theta_1 - \theta_2)\,m_2\,({\omega_2}^2\,L_2 + {\omega_1}^2\,L_1\cos(\theta_1 - \theta_2))}{L_1\,(2\,m_1 + m_2 - m_2\cos(2\,\theta_1 - 2\,\theta_2))}$$

$$\omega_2' = \frac{2\sin(\theta_1 - \theta_2)\,({\omega_1}^2\,L_1\,(m_1 + m_2) + g(m_1 + m_2)\cos\theta_1 + {\omega_2}^2\,L_2\,m_2\cos(\theta_1 - \theta_2))}{L_2\,(2\,m_1 + m_2 - m_2\cos(2\,\theta_1 - 2\,\theta_2))}$$

(1) Double Pendulum Chaos Demonstration - YouTube

LORENTZOV ATRAKTOR



“PROSIM PRERAČUNAJ IN POVEJ MAMI, DA SE TEGA NE DA NAZAJ UREDITI.”


HVALA ZA POZORNOST!