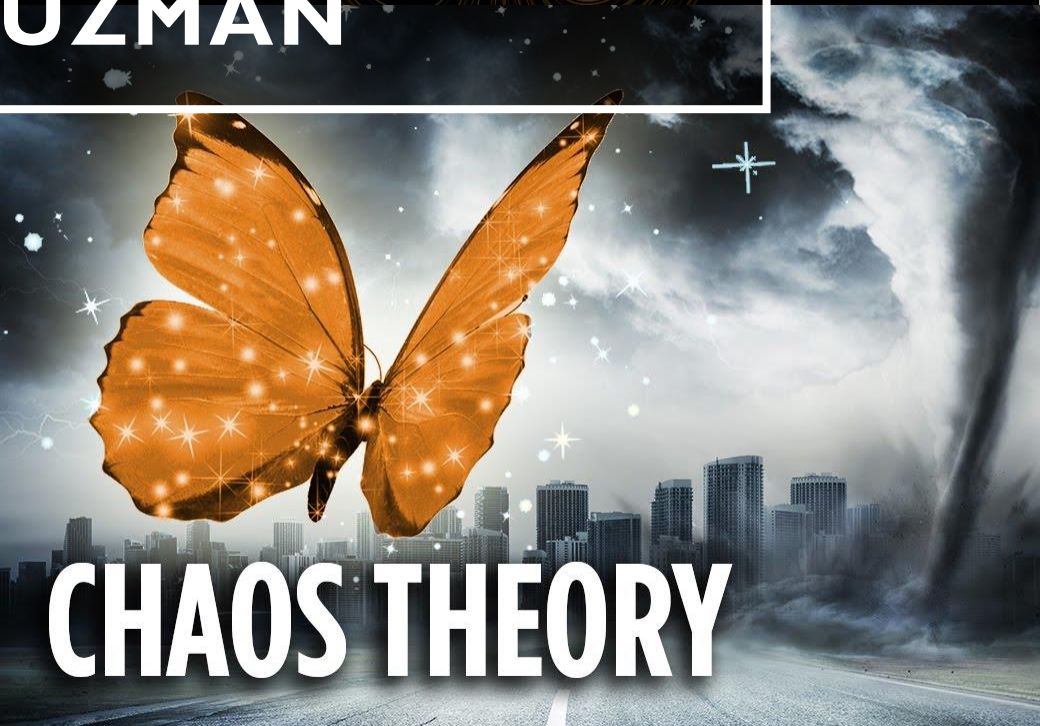
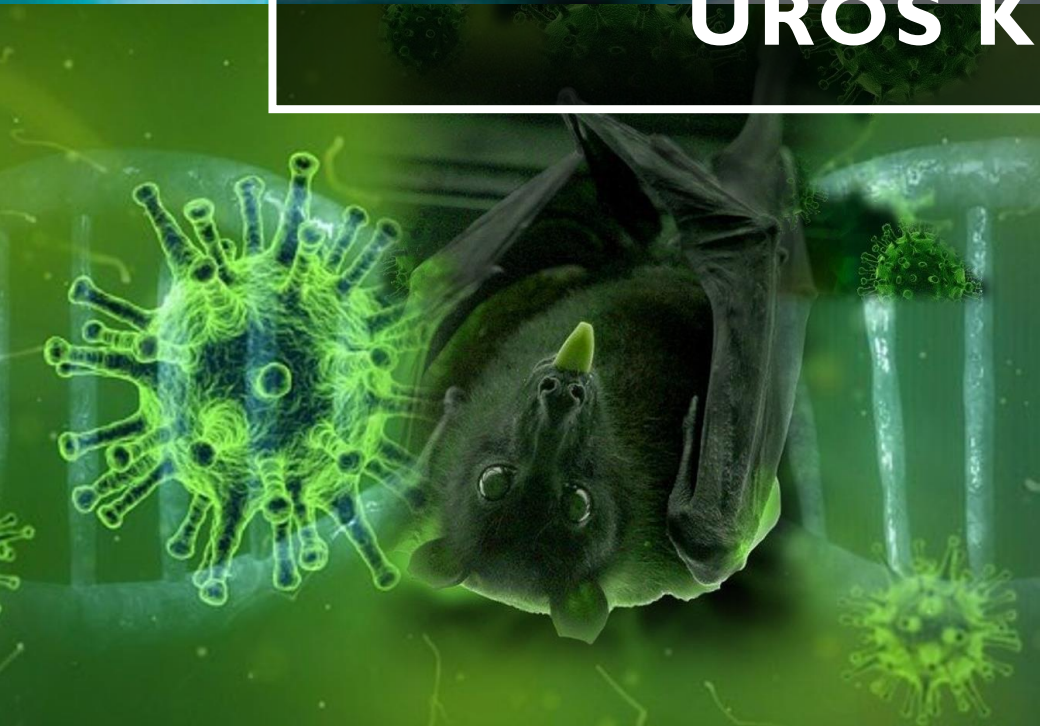




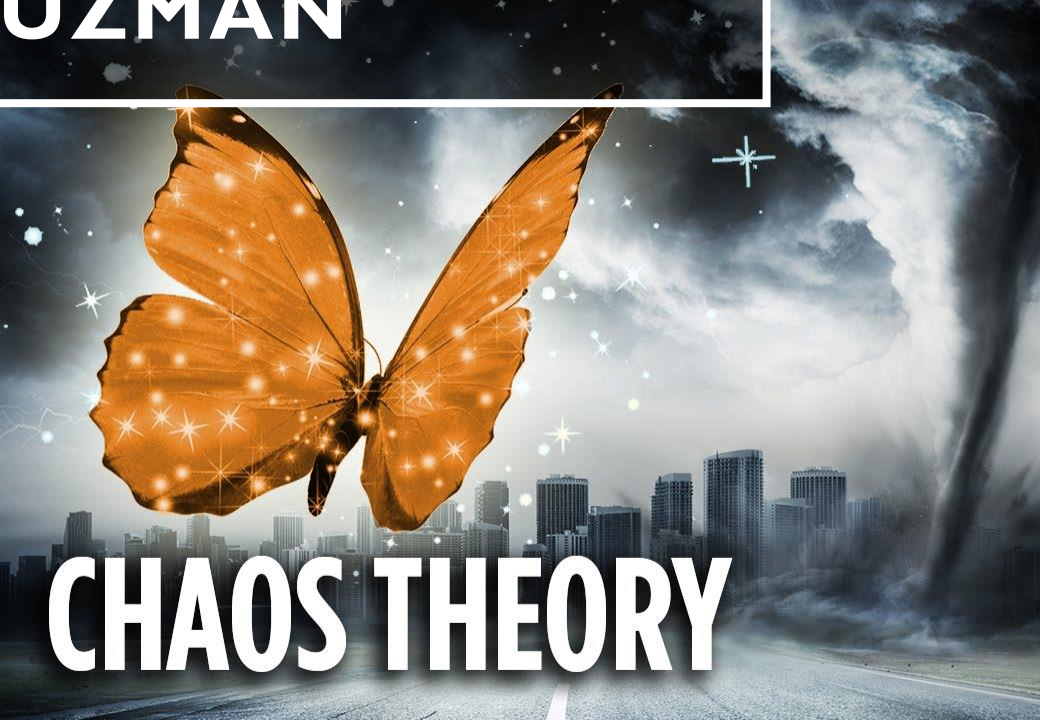
**UČINEK METULJA
UROŠ KUZMAN**



CHAOS THEORY



NETOPIRJA
UČINEK METULJA
UROŠ KUZMAN



CHAOS THEORY



SHERLOCK HOLMES

TAKOJ

PO 1h

JEZERO

9°C

7°C

5°C

Kdaj se je zgodil umor?

***OHLAJANJE JE SORAZMERNO S
TEMPERATURNO RAZLIKO!***

$$T_n = T_{n-1} - k(T_{n-1} - T_j), k > 0$$

**REKURZIVNO
ZAPOREDJE**

NAŠI PODATKI: $T_0 = 9^\circ\text{C}$, $T_1 = 7^\circ\text{C}$, $T_j = 5^\circ\text{C}$

IŠČEMO: $k > 0$ in $n < 0$, da bo $T_n = 37^\circ\text{C}$.

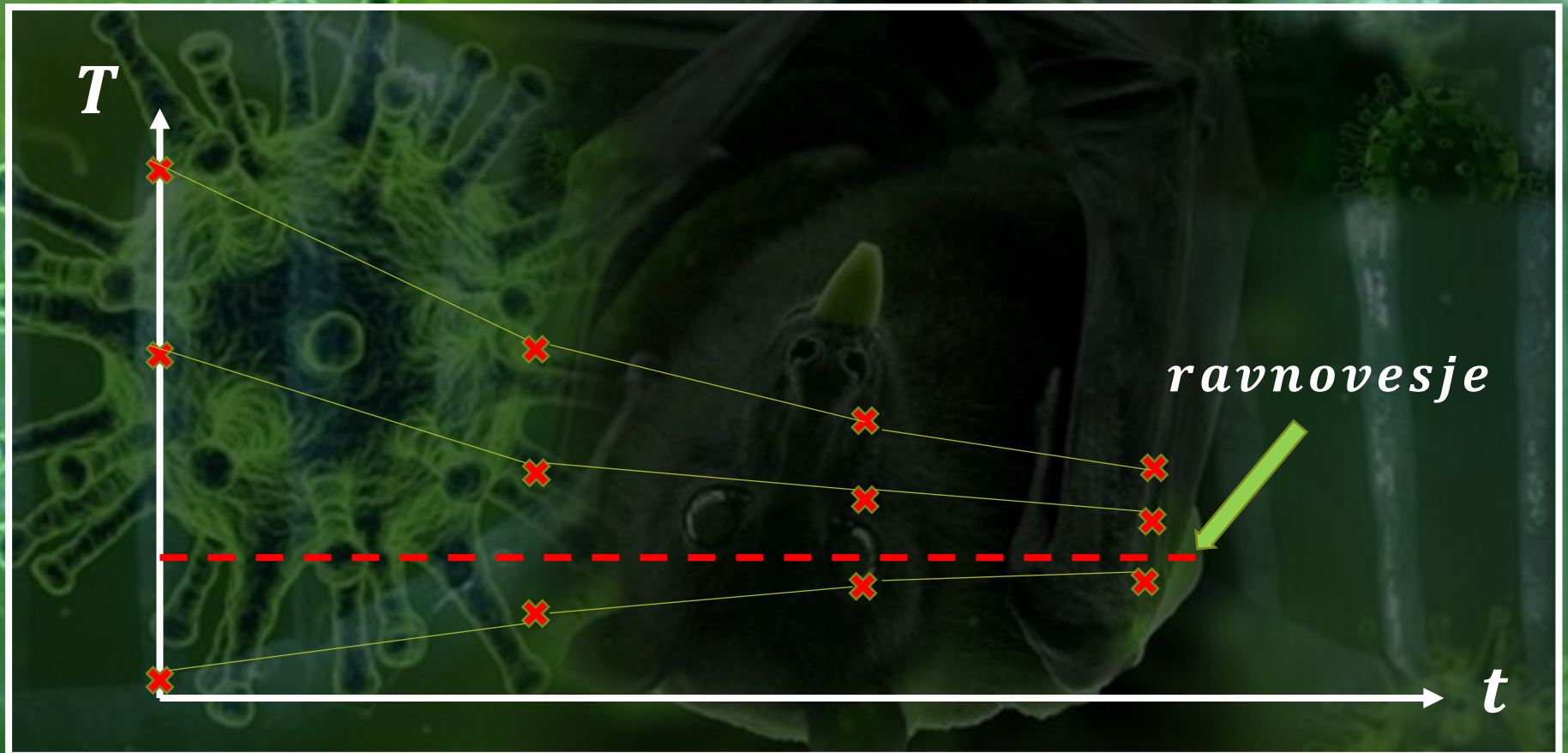
SHERLOCK HOLMES

$$T_n = \frac{1}{2}T_{n-1} + \frac{5}{2}^{\circ}\text{C}$$

**FIKSNA OZ.
RAVNOVESNA
TOČKA $T_0 = 5^{\circ}\text{C}$**

EKSPPLICITNA REŠITEV
$$T_n = \left(\frac{1}{2}\right)^n (T_0 - 5^{\circ}\text{C}) + 5^{\circ}\text{C}$$

SHERLOCK HOLMES



SHERLOCK HOLMES





*KAJ JE NASPROTJE “PREDVIDLJIVOSTI”
OZ. KDAJ SE NEKO REKURZIVNO
ZAPOREDJE OBNAŠA “KAOTIČNO”?*

DEFINICIJA KAOSA

$$f: I \subseteq \mathbb{R} \rightarrow I$$
$$x_n = f(x_{n-1})$$

$$f^{m+k}(x_0) = f^m(x_0) \text{ za}$$

neka $m \in \mathbb{N}_0$ in $k \in \mathbb{N}$.

TRIJE POGOJI:

- *Gostost periodičnih točk*; Periodične točke funkcije f so goste na I .
- *Tranzitivnost*; Za poljubna odprta podintervala $U_1 \in I$ in $U_2 \in I$ obstaja točka $x_0 \in U_1$ in $n > 0$ tako, da velja $f^n(x_0) \in U_2$.
- *Občutljivostna konstanta*; Obstaja $\beta > 0$, da za vsak $x_0 \in I$ in vsak odprt interval U okrog točke x_0 obstajata točka $y_0 \in U$ in $n > 0$, da velja $|f^n(x_0) - f^n(y_0)| > \beta$.

TEJ LASTNOSTI PRAVIMO UČINEK METULJA

POENOSTAVLJENO

*UČINEK METULJA POMENI, DA IMAMO
POLJUBNO BLIZU ZAČETNE TOČKE TUDI
TAKO, KI SE BO "OBNAŠALA POVSEM
DRUGAČE" OZ. BO NJENO ZAPOREDJE
"ŠLO DALEČ STRAN OD OSNOVNEGA"!*

PRIMER: PODVOJITVENA PRESLIKAVA

$$f: [0, 1) \rightarrow [0, 1)$$
$$f(x) = 2x - \lfloor 2x \rfloor$$

**ŠTEVILO PODVOJIMO IN MU
ODŠTEJEMO 1, ČE JE TREBA.**

DOKAŽI: “UČINEK METULJA”

The background of the slide is a microscopic image showing various biological structures. There are green, fuzzy, spherical structures at the top and bottom, and blue, elongated, tube-like structures in the middle. The overall color palette is green and blue.

NAMIG

BINARNI DECIMALNI ZAPIS!

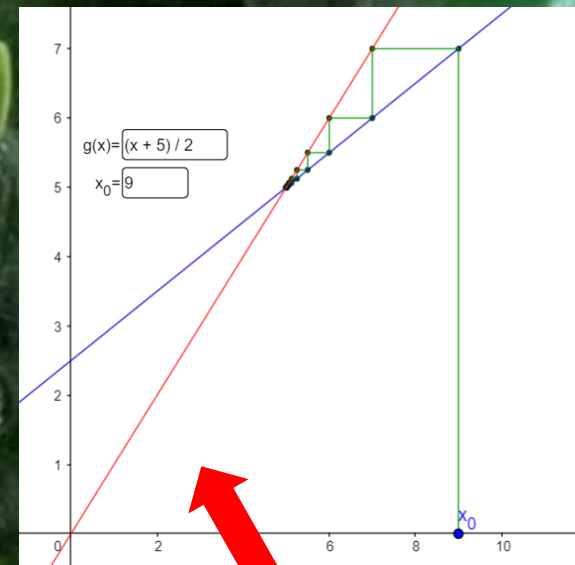
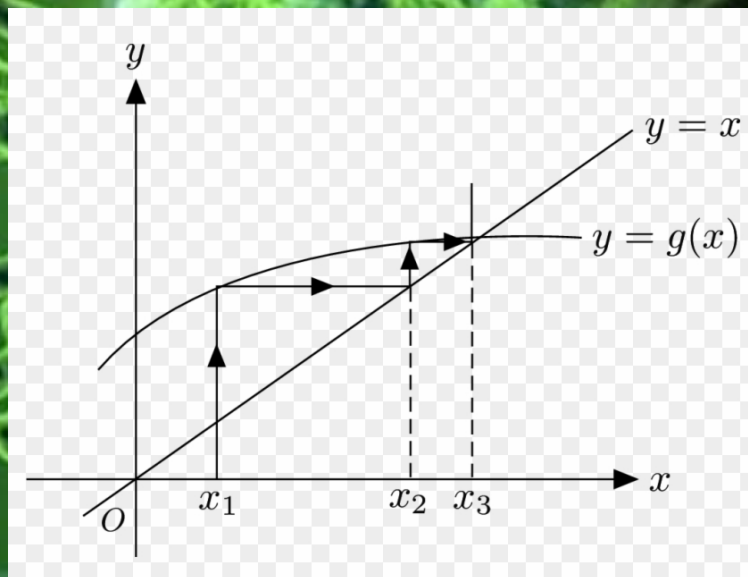
The background of the slide is a complex, microscopic image. It features various green and blue structures, including what appear to be biological cells, fibers, and possibly a small, dark, winged insect-like structure in the upper right. The overall color palette is dominated by shades of green and blue, with some red and white highlights from the text boxes.

DODATNO VPRAŠANJE

*ALI ZNAMO DOKAZATI TUDI DRUGI DVE
LASTNOSTI IZ DEFINICIJE KAOSA?*

PAJČEVINASTI DIAGRAM

PRI VIZUALIZACIJI RAZVOJA ZAPOREDJA SI LAHKO POMAGAMO Z GRAFOM FUNKCIJE.

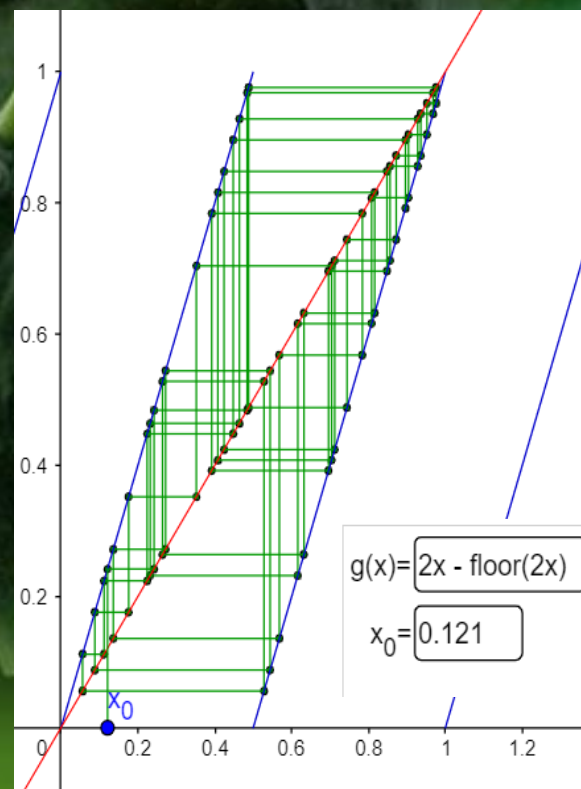
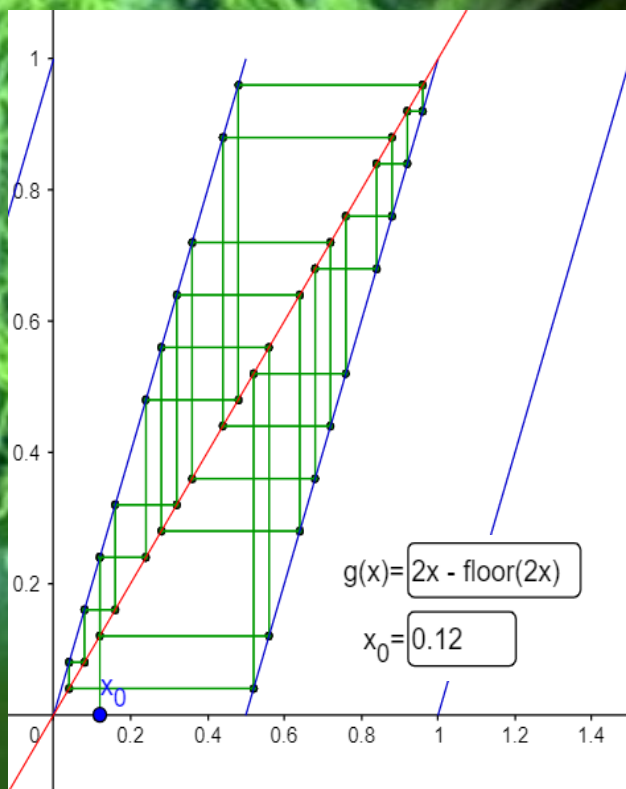


Cobweb plotter – GeoGebra

SHERLOCK

PRIMER: PODVOJITVENA PRESLIKAVA

$$f: [0, 1) \rightarrow [0, 1)$$
$$f(x) = 2x - \lfloor 2x \rfloor$$

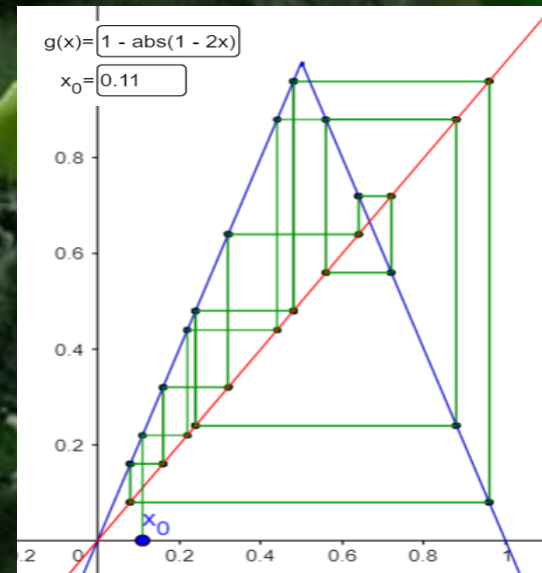
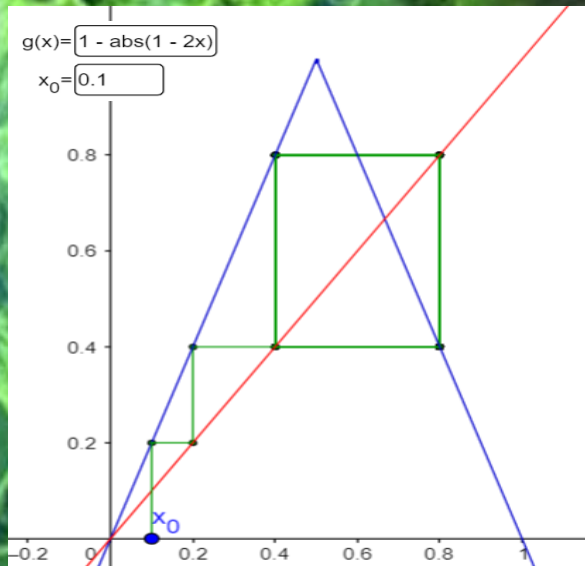


The image is a composite graphic. In the background, there is a dark bat with its wings spread, set against a green, textured background that resembles a forest or a microscopic environment. Overlaid on the bat is a large, glowing green virus particle with a red core and numerous spikes. A prominent red banner with a white border is positioned horizontally across the center of the image. Inside this banner, the text 'DOMAČA NALOGA' is written in a white, italicized, serif font.

DOMAČA NALOGA

ŠOTORSKA PRESLIKAVA

$$f: [0, 1) \rightarrow [0, 1)$$
$$f(x) = 1 - |1 - 2x| = 2 \min(x, 1 - x)$$

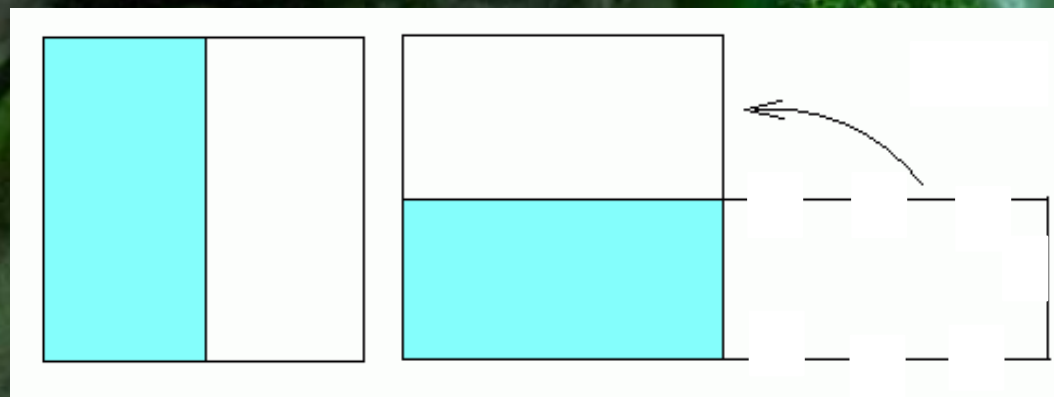


**Z BINOMSKIM ZAPISOM DOKAŽI, DA JE
TA PRESLIKAVA KAOTIČNA!**



“UPORABEN” PRIMER

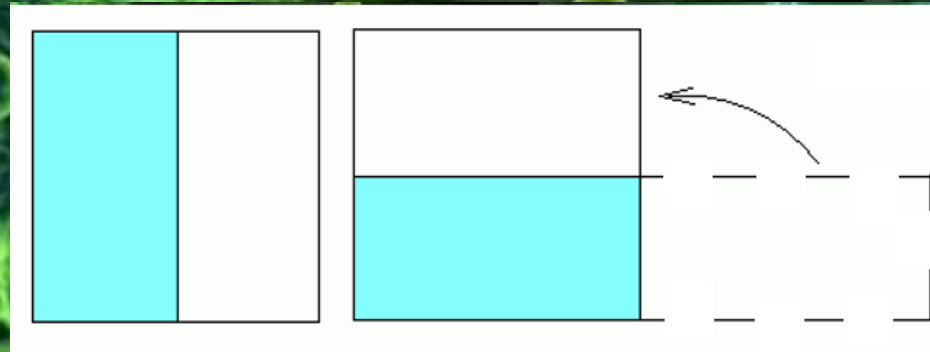
PRIMER: PEKOVA PRESLIKAVA



**VIŠINO TESTA RAZPOLOVIMO, GA
PREREZEMO NA POL IN DESNO
POLOVICO POLOZOMO NA LEVO.**

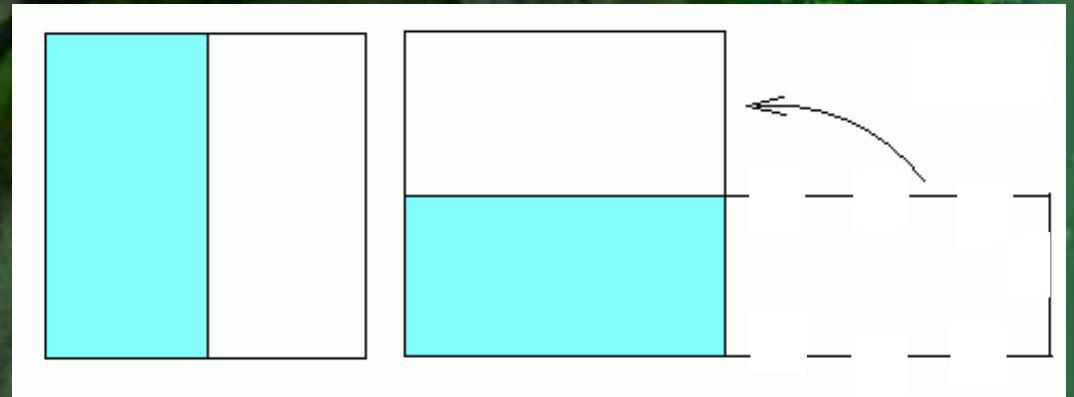
PRIMER: PEKOVA PRESLIKAVA

$$f: [0, 1) \times [0, 1) \rightarrow [0, 1) \times [0, 1)$$
$$f(x, y) = (2x - [2x], \frac{1}{2}(y + [2x]))$$



DOKAŽIMO “UČINEK METULJA”!

PRIMER: PEKOVA PRESLIKAVA



KAKO TA KAOS IZGLEDA V PRAKSI:
[Baker's map mixing - Baker's map - Wikipedia](#)

The background is a composite image with a green tint. In the center, a bat is shown with its wings spread. To the left, a hand is depicted with several glowing green spots, resembling a virus or a pathogen. The overall theme suggests a connection between wildlife, human health, and infectious diseases.

TERMINOLOŠKI SLOVAR

TERMINOLOŠKI SLOVAR

$$f: S \rightarrow S$$
$$x_n = f(x_{n-1})$$

n - naravno
ali celo število

S - množica stanj, ki
ni nujno v \mathbb{R}^n

**TAKIM ZAPOREDJEM PRAVIMO TUDI
DISKRETNi DINAMIČNI SISTEMI**

**V SPLOŠNEM: DINAMIČNI SISTEM POMENI PRAVILO,
KI POVE, KAKO SE “SPREMINJA” NEKO STANJE.**

KOMENTAR

***KAOS SE NE ZGODI "PO NAKLJUČJU",
AMPAK OB VNAPREJ DOLOČENIH
PRAVILIH (DINAMIČNI SISTEMI SO
DETERMINISTIČNI)!***

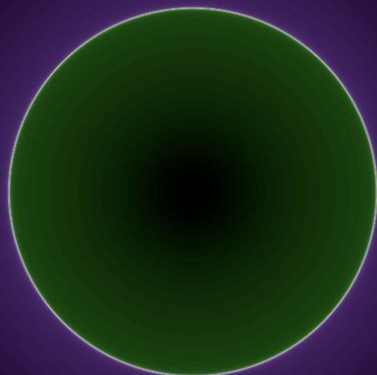


*ŠE NEKAJ ZANIMIVOSTI
IZ TEORIJE KAOSA!*

IZREK ŠARKOVSKEGA

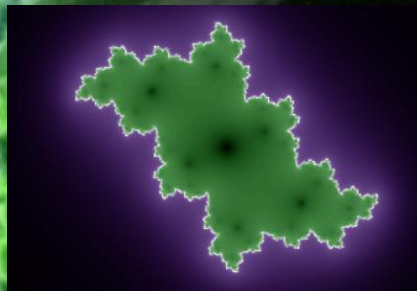
*ČE IMA DISKRETNI DINAMIČNI SISTEM
NA ZAPRTEM INTERVALU
3-PERIODIČNO REŠITEV, IMA TUDI
REŠITVE VSEH OSTALIH PERIOD.*

KOMPLEKSNA DINAMIKA

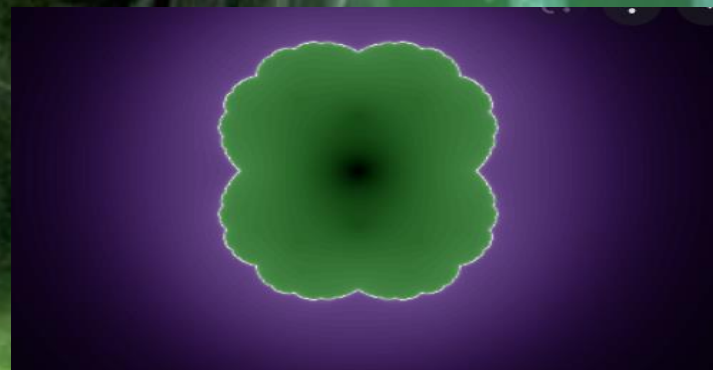
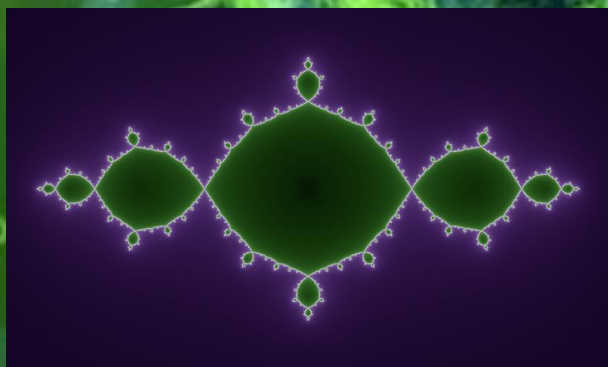


$$f: \mathbb{C} \rightarrow \mathbb{C}$$
$$f(z) = z^2$$

KOMPLEKSNA DINAMIKA



$$f: \mathbb{C} \rightarrow \mathbb{C}$$
$$f(z) = z^2 + c$$



PROBLEM IZ VIDIKA FIZIKA ...

*ČAS NI DISKRETNA
SPREMENLJIVKA!*



REŠITEV



OD DISKRETNIH K ZVEZNIM SISTEMOM

$$f: S \rightarrow S$$
$$x_n = f(x_{n-1})$$

n naravno ali
celo število

$$f: S \rightarrow S$$
$$\dot{x} = f(x)$$

t realno število
oz. čas

**ZVEZNI DINAMIČNI SISTEM JE PODAN Z
AVTONOMNO DIFERENCIALNO ENAČBO!**

VPRAŠANJE

**KAKO VSAJ DELČEK TE TEORIJE
OBRAVNAVATI S SREDNJEŠOLCI?**

ZVEZNI DINAMIČNI SISTEMI



**Kako skakalcu izračunajo
hitrost na odskočni mizi?**

ZVEZNI DINAMIČNI SISTEMI



$$v = \frac{\Delta s}{\Delta t} \text{ za zelo majhen } \Delta t$$

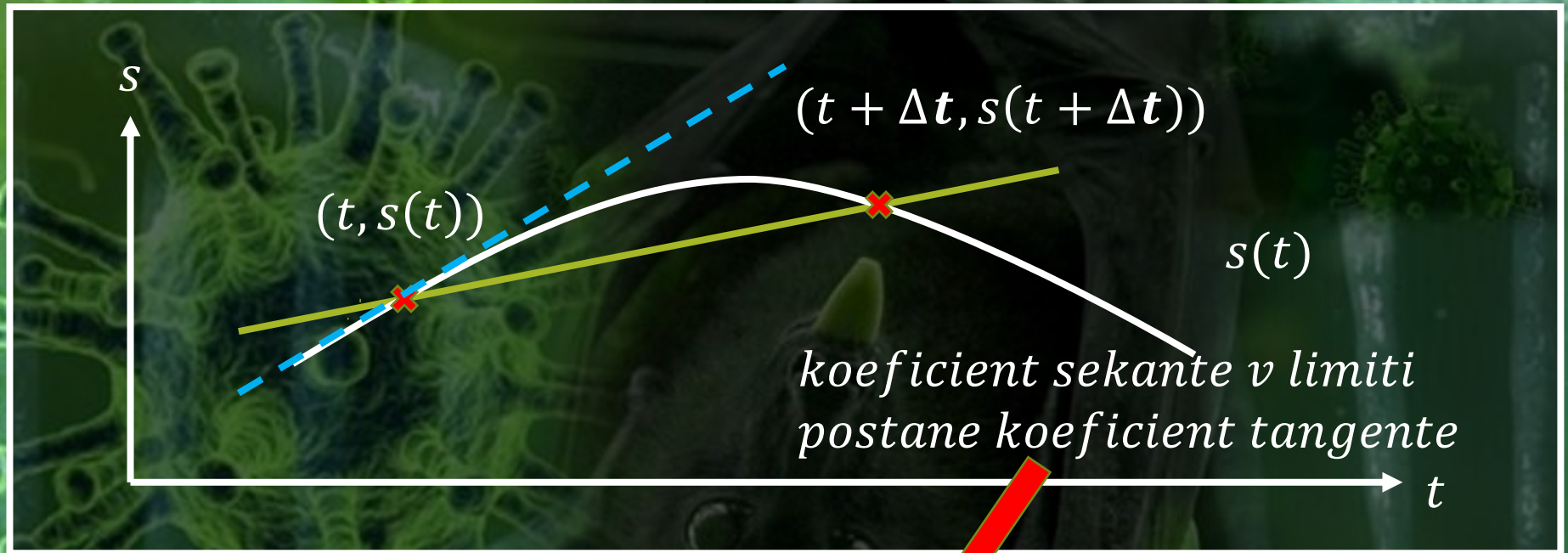
ZVEZNI DINAMIČNI SISTEMI



odvod

$$v(t) = \lim_{\Delta t \rightarrow 0} \frac{s(t + \Delta t) - s(t)}{\Delta t} = \dot{s}(t)$$

GEOMETRIJSKI POMEN ODVODA



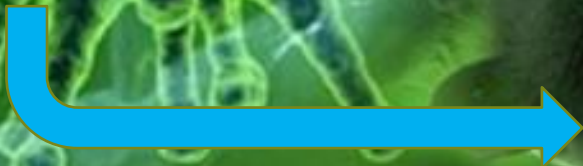
$$v(t) = \lim_{\Delta t \rightarrow 0} \frac{s(t + \Delta t) - s(t)}{\Delta t} = \dot{s}(t)$$



“ZVEZNI” SHERLOCK HOLMES

$$T_n = T_{n-1} - \frac{1}{2}(T_{n-1} - T_j)$$

HITROST SPREMINJANJA
TEMPERATURE JE
SORAZMERN
TEMPERATURNI RAZLIKI



$$\dot{T}(t) = -\frac{1}{2}(T(t) - T_j)$$



**ALI ZNAMO
TAKO
ENAČBO
REŠITI?**

$$\dot{T}(t) = -\frac{1}{2}(T(t) - T_j)$$

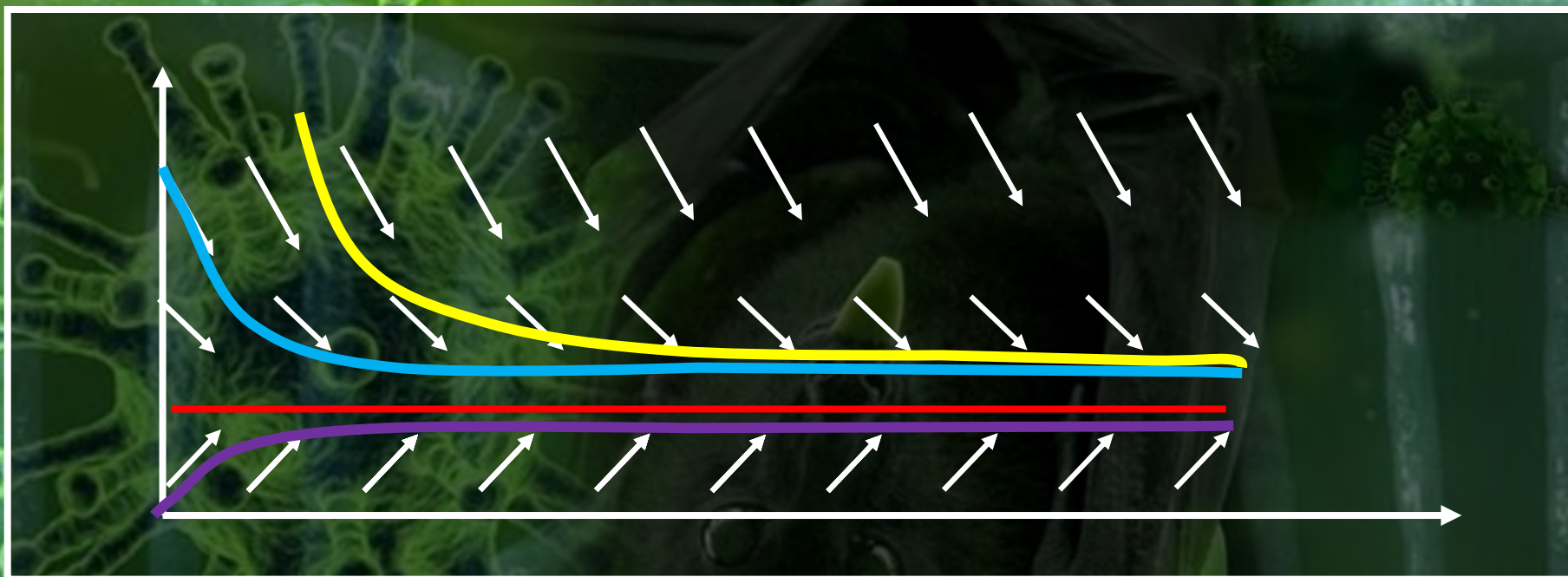
**ODGOVOR: EKSPPLICITNO ŠELE NA FMF!
IMAMO PA NA VOLJO "GRAFIČNO METODO".**

ZVEZNI SHERLOCK HOLMES



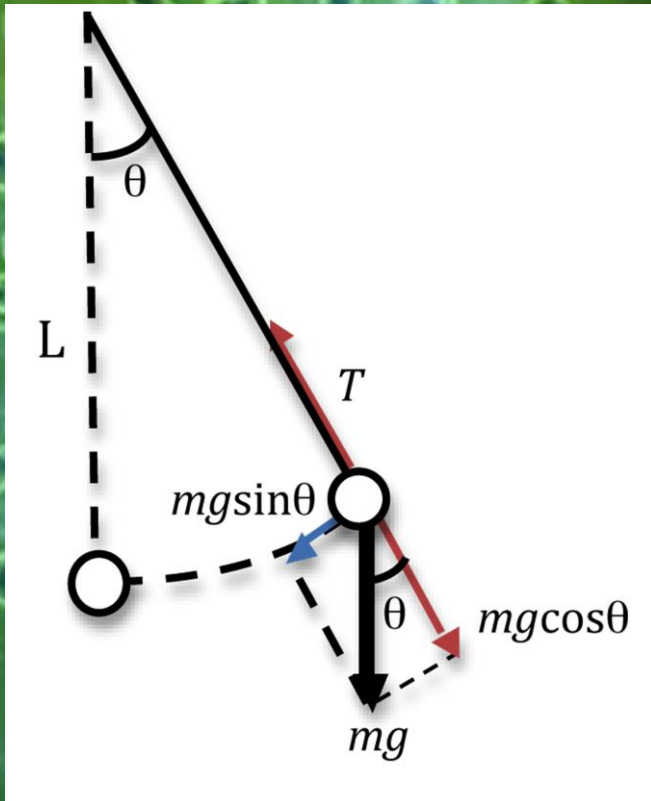
$$\dot{T}(t) = -\frac{1}{2}(T(t) - T_j)$$

ZVEZNI SHERLOCK HOLMES



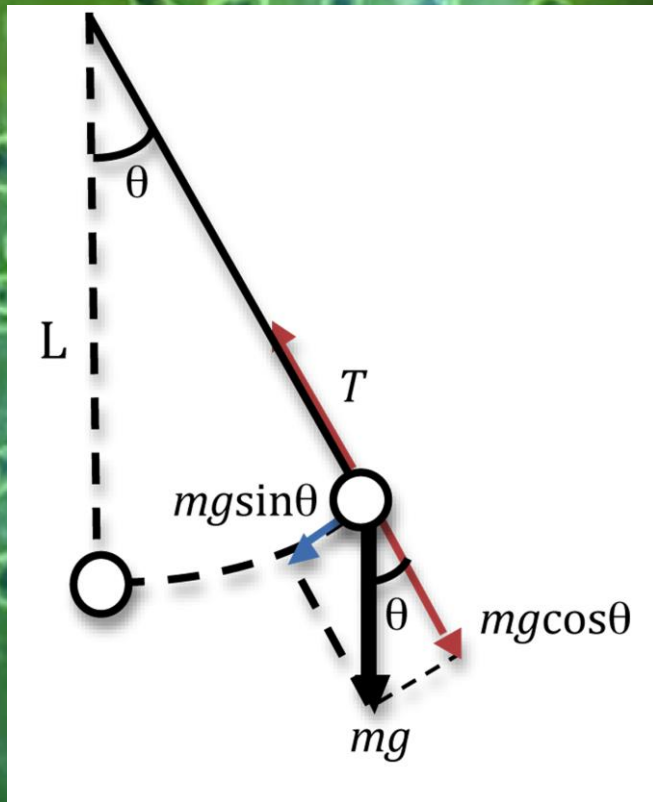
EKSPPLICITNO: $T(t) = \left(\frac{1}{2}\right)^t (T_0 - T_j) + T_j$

PRIMER: NIHALO



$$F = ma = -mg \sin(\theta)$$

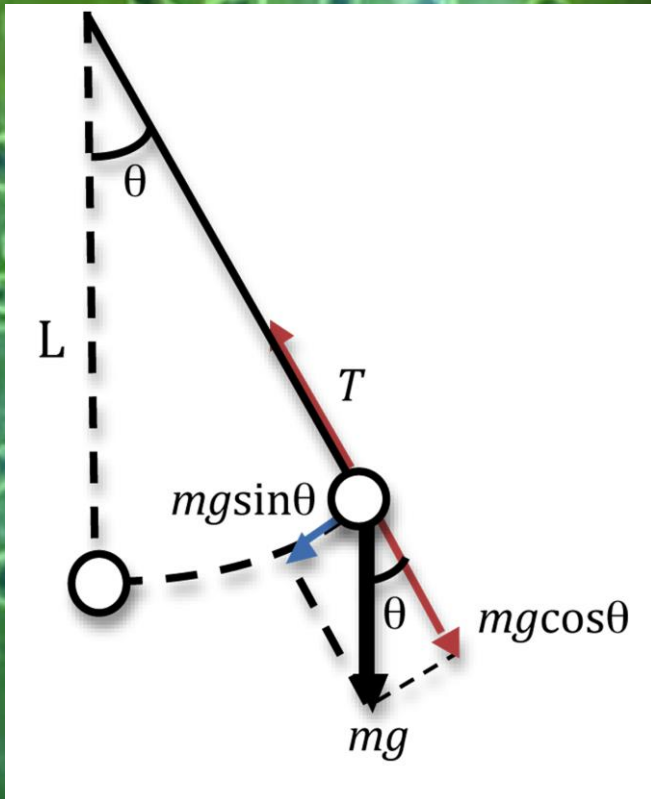
PRIMER: NIHALO



$$F = ma = -mg \sin(\theta)$$

**POSPEŠEK POMENI,
KAKO HITRO SE
SPREMINJA HITROST.**

PRIMER: NIHALO

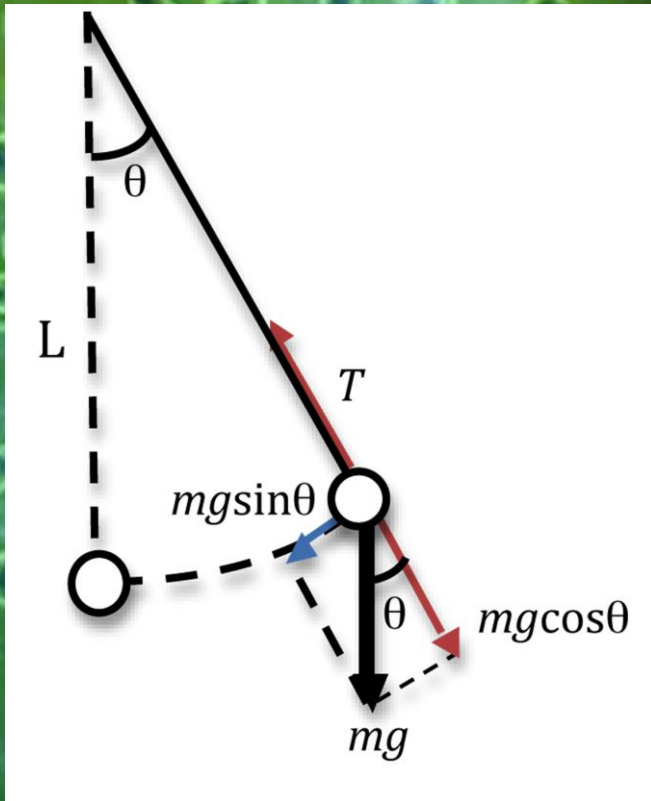


$$F = ma = -mg \sin(\theta)$$

**POSPEŠEK POMENI,
KAKO HITRO SE
SPREMINJA HITROST.**

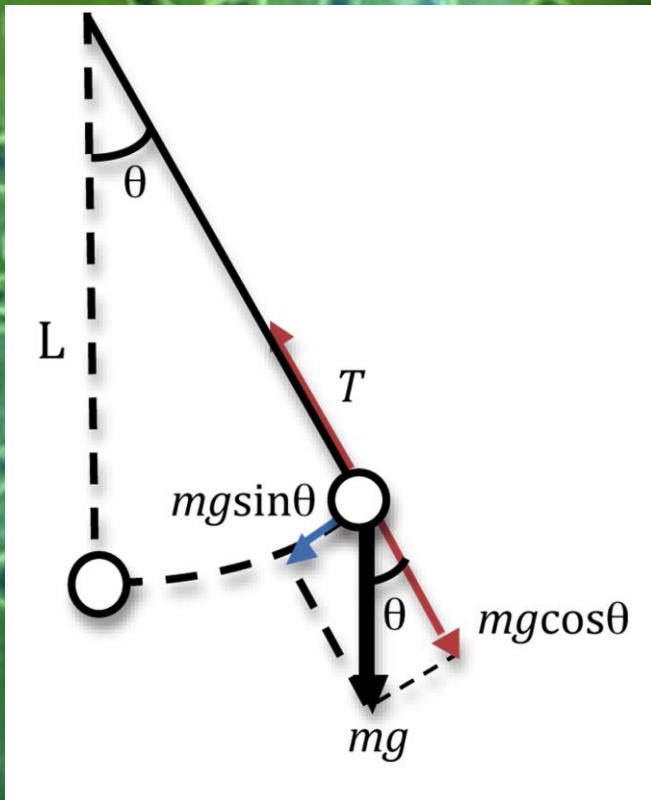
$$a = \dot{\omega}$$

PRIMER: NIHALO



$$F = \cancel{m\dot{\omega}} = -\cancel{mg} \sin(\theta)$$

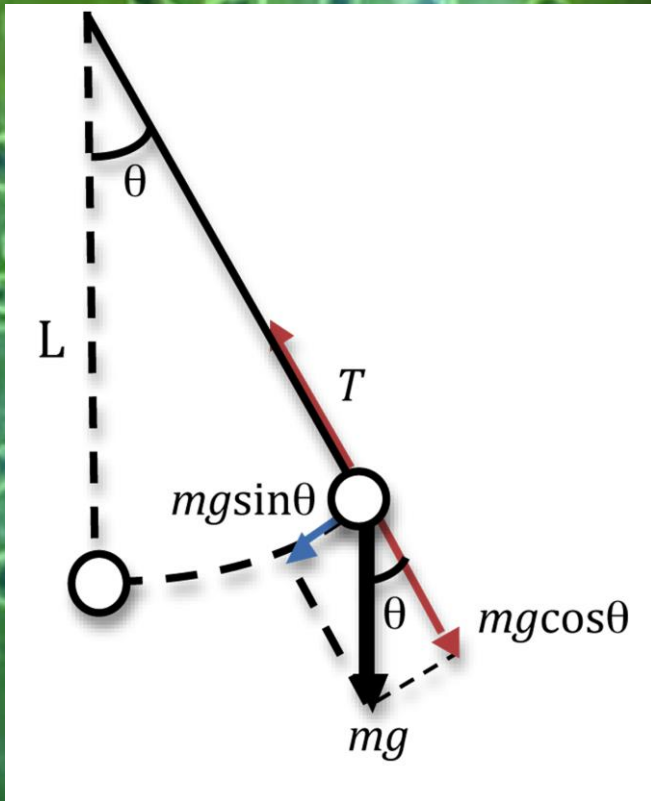
PRIMER: NIHALO



$$F = \cancel{m\dot{\omega}} = -\cancel{mg} \sin(\theta)$$

**DODAMO ŠE
ENAČBO $\dot{\theta} = \omega$.**

PRIMER: NIHALO



$$F = \cancel{m\dot{\omega}} = -\cancel{mg} \sin(\theta)$$

DODAMO ŠE
ENAČBO $\dot{\theta} = \omega$.

$$\dot{\theta} = \omega$$
$$\dot{\omega} = -g \sin(\theta)$$

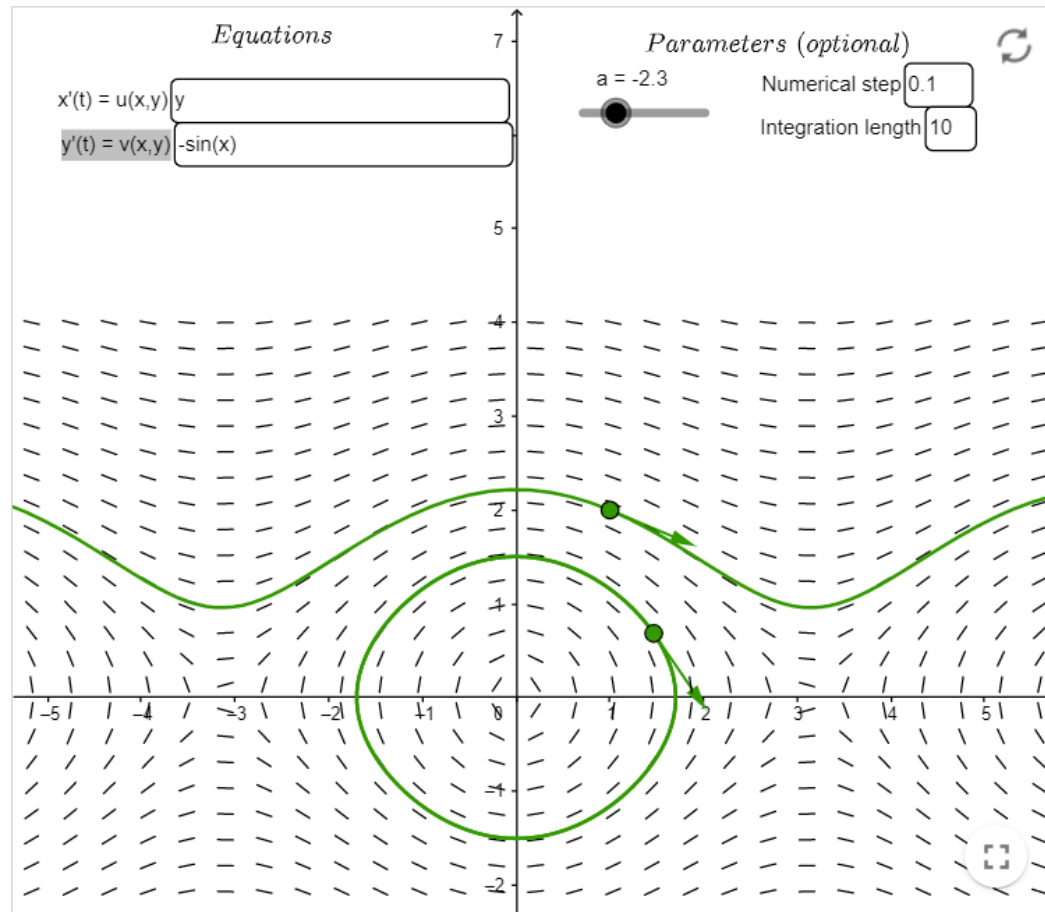
GEOMETRIJSKI POMEN



(\dot{x}, \dot{y}) pomeni tangентno smer

PRIMER: NIHALO

Phase plane – GeoGebra



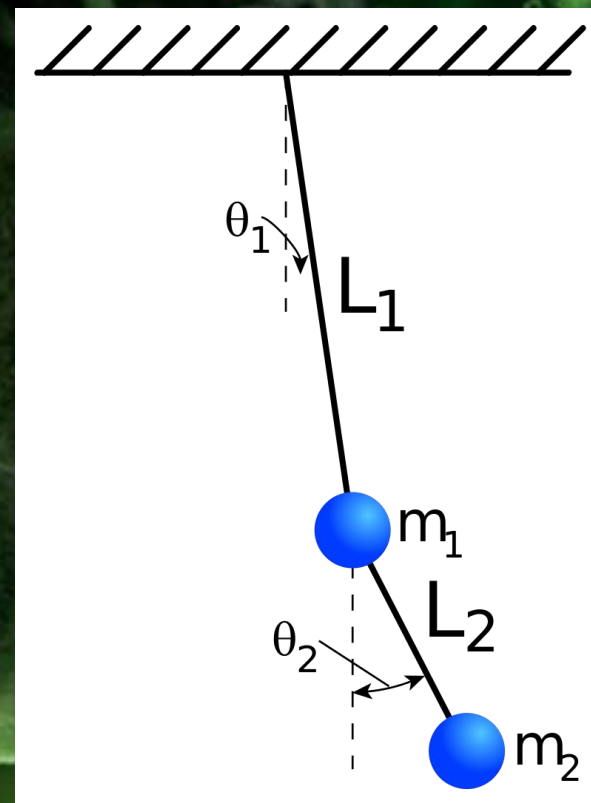
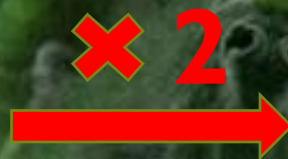
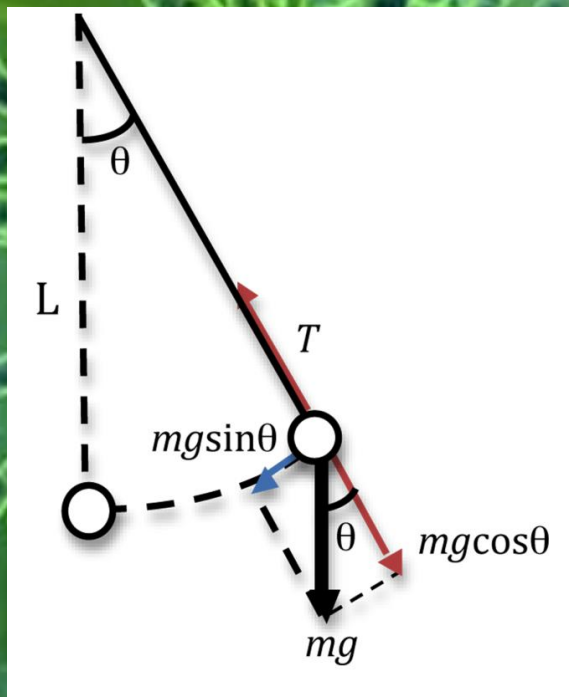
PRIMER: NIHALO



PREDVIDLJIVO

**KAOS SE PRI "LEPIH"
ZVEZNIH DINAMIČNIH
SISTEMIH POJAVI ŠELE V
DIMENZIJI ≥ 3 !**

REŠITEV: PODVOJIMO DIMENZIJO



PRIMER: DVOJNO NIHALO

$$\theta_1' = \omega_1$$

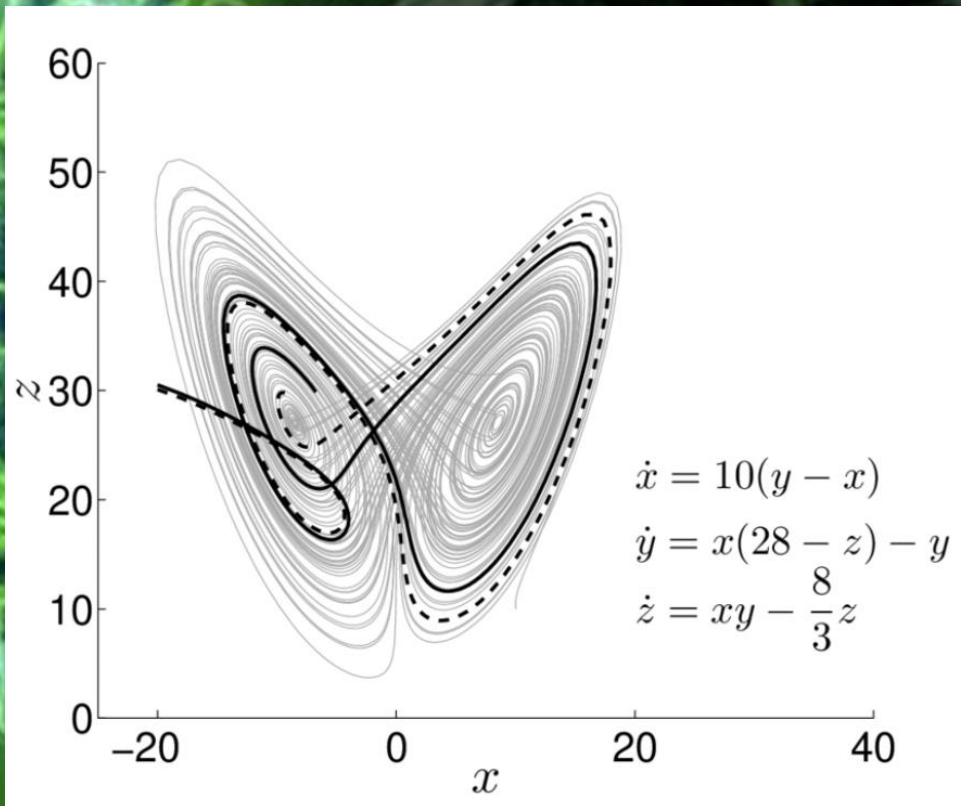
$$\theta_2' = \omega_2$$

$$\omega_1' = \frac{-g(2m_1 + m_2) \sin \theta_1 - m_2 g \sin(\theta_1 - 2\theta_2) - 2 \sin(\theta_1 - \theta_2) m_2 (\omega_2^2 L_2 + \omega_1^2 L_1 \cos(\theta_1 - \theta_2))}{L_1 (2m_1 + m_2 - m_2 \cos(2\theta_1 - 2\theta_2))}$$

$$\omega_2' = \frac{2 \sin(\theta_1 - \theta_2) (\omega_1^2 L_1 (m_1 + m_2) + g(m_1 + m_2) \cos \theta_1 + \omega_2^2 L_2 m_2 \cos(\theta_1 - \theta_2))}{L_2 (2m_1 + m_2 - m_2 \cos(2\theta_1 - 2\theta_2))}$$

[\(1\) Double Pendulum Chaos Demonstration - YouTube](#)

LORENTZOV ATRAKTOR







*“PROSIM PRERAČUNAJ IN POVEJ MAMI,
DA SE TEGA NE DA NAZAJ UREDITI.”*

The background is a vibrant green with a dark, textured appearance. In the center, a bat is shown from a top-down perspective, its wings partially spread. Below the bat, a mouse is visible, looking upwards. To the left of the mouse, there is a large, glowing green virus particle with a complex, multi-lobed structure. The overall scene suggests a connection between these animals and the virus.

HVALA ZA POZORNOST!