

Nekatero matematične funkcije v fiziki

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Uvod

- pomen matematike pri razumevanju fizike
- kaj je funkcija?
 - $f: A \rightarrow B$
 - $f: a \mapsto b, b = f(a)$
- graf funkcije f je množica urejenih parov (a, b) in je podmnožica kartezičnega produkta $A \times B$.
- fiziki funkcije uporabljamo kot zveze med količinami – ugotovljene z opazovanji (zakoni), izpeljane iz njih (izreki) ali definirane

Merjenje v fiziki

$$\bar{x} = \frac{\sum_{i=1}^N x_i}{N} = \frac{x_1 + x_2 + \dots + x_N}{N}$$

$$x = \bar{x} \pm \Delta x$$

$$x = \bar{x} \left(1 \pm \frac{\Delta x}{\bar{x}} \right) = \bar{x} (1 \pm \delta x)$$

$$(x_1 \pm \Delta x_1) \pm (x_2 \pm \Delta x_2) = (x_1 + x_2) \pm (\Delta x_1 + \Delta x_2)$$

$$(x_1 \pm \Delta x_1) \cdot (x_2 \pm \Delta x_2) = x_1 \cdot x_2 (1 \pm (\delta x_1 + \delta x_2))$$

$$\frac{(x_1 \pm \Delta x_1)}{(x_2 \pm \Delta x_2)} = \frac{x_1}{x_2} (1 \pm (\delta x_1 + \delta x_2))$$

Opis premega gibanja

$$\bar{v} = \frac{s}{t}$$

$$\vec{v} = \frac{\Delta \vec{x}}{\Delta t}$$

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

$$x(t) = x_0 + v_0 t + \frac{at^2}{2}$$

$$v(t) = v_0 + at$$

$$v^2(t) = v_0^2 + 2as$$

Site in gibanje

$$\vec{a} = \frac{\Sigma \vec{F}}{m}$$

$$F = kx$$

$$F_{tr} = k_{tr} F_N; \quad F_{l(max)} = k_l F_N$$

$$p_{tek} = \rho gh$$

$$F_{vzg} = \rho_{tek} g V_{pot}$$

$$\rho = \frac{m}{V}$$

$$p = \frac{F}{S}$$

$$v = \frac{l}{t_0}$$

$$\omega = \frac{\Delta \alpha}{\Delta t}$$

$$v = \frac{2\pi r}{t_0}$$

$$a_r = \frac{v^2}{r}$$

$$F_g = \frac{Gm_1 m_2}{r^2}$$

$$g = \frac{Gm}{r^2}$$

$$g(r) = g \frac{r_z^2}{r^2}$$

$$\vec{M} = \vec{F} \times \vec{r} \text{ ali } M = Fr, \quad F \perp r$$

Izrek o gibalni količini

$$\vec{G} = m\vec{v}$$

$$\vec{F} \Delta t = m\Delta \vec{v}$$

$$l: \vec{F}_c = \Phi_m \Delta \vec{v}$$

Delo in energija

$$A = FS, \quad \vec{F} \parallel \vec{s}$$

$$A = \vec{F} \cdot \Delta \vec{x} = |\vec{F}| |\Delta \vec{x}| \cos \alpha$$

$$A = -p\Delta V$$

$$P = \frac{A}{t} = \vec{F} \cdot \vec{v}$$

$$W_k = \frac{mv^2}{2}$$

$$\Delta W_p = mg\Delta h$$

$$l: W_p = -\frac{Gm_1 m_2}{r}$$

$$W_{pr} = \frac{kx^2}{2}$$

Zgradba snovi in temperatura

$$n = \frac{m}{M} = \frac{N}{N_A}$$

$$\Delta l = \alpha \Delta T$$

$$\Delta V = V\beta \Delta T$$

$$pV = nRT,$$

Vrtenje togega telesa

$$l: \alpha = \frac{\Delta \omega}{\Delta t}$$

$$l: J = mr^2$$

$$l: \alpha = \frac{M}{J}$$

$$l: J_{valj} = \frac{1}{2} mr^2$$

$$l: J_{krogla} = \frac{2}{5} mr^2$$

$$l: J_{patika} = \frac{1}{12} ml^2$$

$$l: \Gamma = J\omega$$

$$l: M\Delta t = J\omega$$

$$l: W_{rot} = \frac{J\omega^2}{2}$$

Tekočine

$$\Phi_V = \frac{\Delta V}{\Delta t}$$

$$\Phi_m = \frac{\Delta m}{\Delta t}$$

$$l: \Phi_V = Sv$$

$$l: p + \rho gh + \frac{1}{2} mv^2 = konst.$$

$$l: F_u = 6\pi r\eta v$$

$$l: F_u = \frac{1}{2} c\rho S v^2$$

$$l: F = \gamma l$$

$$l: \frac{\Delta V}{V} = -\chi \Delta p$$

Notranja energija in toplota

$$A + Q = \Delta W$$

$$A = -p\Delta V$$

$$Q = mc\Delta T$$

$$Q = mq_i$$

$$P = \frac{Q}{t}$$

$$P = \lambda S \frac{\Delta T}{d}$$

$$j = \frac{P}{S} = \sigma T^4$$

$$\eta = \frac{A_{koristno}}{Q_{vložena}}$$

$$W_n = \frac{3}{2} k_B T$$

Električni tok

$$I = \frac{\Delta e}{\Delta t}$$

$$I = \frac{U}{R}$$

$$R = \frac{\xi l}{S}$$

$$\sum_{i=1}^N I_i = 0 \quad \text{in} \quad \sum_{i=1}^N U_i = 0$$

$$R_n = R_1 + R_2 \quad \text{oz.} \quad \frac{1}{R_n} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$P = UI = U_{ef} I_{ef}$$

$$U_{ef} = \frac{U_0}{\sqrt{2}} \quad \text{in} \quad I_{ef} = \frac{I_0}{\sqrt{2}}$$

Električni naboj in električno polje

$$F_e = \frac{e_1 e_2}{4\pi \epsilon_0 r^2}$$

$$\vec{E} = \frac{\vec{F}_e}{e}$$

$$E = \frac{e}{4\pi \epsilon_0 r^2}$$

$$E = \frac{e}{2S\epsilon_0}$$

$$A = e\vec{E}\vec{s} = eU$$

$$W_e = \frac{e_1 e_2}{4\pi \epsilon_0 r}$$

$$l: V = \frac{W_e}{e} = Ez$$

$$l: V = \frac{e}{4\pi \epsilon_0 r}$$

$$l: U_{12} = V_2 - V_1$$

$$l: \epsilon_0 E_1 S = e$$

$$l: W_e = \frac{W_e}{V} = \frac{1}{2} \epsilon_0 E^2$$

$$e = CU$$

$$C = \frac{\epsilon_0 S}{d}$$

$$W_C = \frac{e^2}{2C} = \frac{1}{2} CU^2$$

Magnetno polje

$$B = \frac{\mu_0 I}{2\pi r}$$

$$B = \frac{\mu_0 N I}{l}$$

$$\vec{F} = e\vec{v} \times \vec{B}$$

$$\vec{F} = I\vec{l} \times \vec{B}$$

$$l: \vec{M} = N I \vec{S} \times \vec{B}$$

$$l: U_H = vBy = -\frac{IB}{ne_0 z} = \frac{R_H IB}{z}$$

Indukcija

$$U_i = l\vec{v} \cdot \vec{B}$$

$$\Phi_m = \vec{B} \cdot \vec{S}$$

$$U_i = -\frac{\Delta\Phi_m}{\Delta t}$$

$$\frac{U_1}{U_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2}$$

$$l: L = \frac{\Phi_m}{I} = \frac{\mu_0 N^2 S}{l}$$

$$l: W_m = \frac{LI^2}{2}$$

$$l: W_m = \frac{B^2}{2\mu_0}$$

$$t_0 = 2\pi\sqrt{LC}$$

$$l: E_0 = cB_0$$

$$l: j = \frac{E_0 B_0}{2\mu_0}$$

Nihanje

$$v = \frac{1}{t_0}$$

$$l: \omega = 2\pi\nu$$

$$x(t) = x_0 \sin(\omega t)$$

$$v(t) = v_0 \cos(\omega t), \quad v_0 = x_0 \omega$$

$$a(t) = -a_0 \sin(\omega t), \quad a_0 = x_0 \omega^2$$

$$t_0 = 2\pi \sqrt{\frac{l}{g}}$$

$$t_0 = 2\pi \sqrt{\frac{m}{k}}$$

Valovanje

$$c = v\lambda$$

$$\frac{\sin\alpha_1}{\sin\alpha_2} = \frac{c_1}{c_2} = \frac{n_2}{n_1}$$

$$d\sin\alpha = N\lambda$$

$$c = \sqrt{\frac{Fd}{m}}$$

$$v = v_0 \left(1 \pm \frac{v}{c}\right)$$

$$v = \frac{v_0}{1 \mp \frac{v}{c}}$$

$$l: \sin\varphi = \frac{c}{v}$$

$$j = \frac{P}{S} = \frac{P}{4\pi r^2}$$

$$l: G = 10 \log_{10} \frac{j}{j_0}$$

Svetloba

$$n = \frac{c_0}{c_{snov}}$$

$$\frac{\sin\alpha_1}{\sin\alpha_2} = \frac{c_1}{c_2} = \frac{n_2}{n_1}$$

$$\frac{1}{f} = \frac{1}{a} + \frac{1}{b}$$

$$M = \frac{S}{P} = \frac{b}{a}$$

$$d\sin\alpha = N\lambda$$

$$j = \frac{P}{S} = \frac{P}{4\pi r^2}$$

Atom

$$m = Au$$

$$W_f = h\nu = \frac{hc}{\lambda}$$

$$W = \frac{-13,6 \text{ eV}}{n^2}$$

$$W_f = A_i + W_k$$

$$l: \lambda_{\min} = \frac{hc}{e_0 U}$$

atomsko jedro

$$r = r_0 A^{1/3}$$

$$W = \Delta mc^2$$

$${}^A_Z X \rightarrow {}^A-4_Z-2 Y + {}^4_2 \alpha$$

$${}^A_Z X \rightarrow {}^A_{Z+1} Y + e^- + \bar{\nu}$$

$${}^A_Z X \rightarrow {}^A_{Z-1} Y + e^+ + \nu$$

$${}^A_Z X \xrightarrow{\gamma} {}^A_Z X + \gamma$$

$$A = \frac{\Delta N}{\Delta t}$$

$$A = N\lambda$$

$$N = N_0 2^{-\frac{t}{t_{1/2}}} = N_0 e^{-\lambda t}$$

$$\lambda = \frac{\ln 2}{t_{1/2}}$$

Astronomija

$$\frac{r^3}{t_0^2} = \text{konst.}$$

$$l: v = H_0 r$$

$$j = \frac{P}{4\pi r^2} = \sigma T^4$$

$$l: \lambda_0 T = k_W$$

Teorija relativnosti

$$l: x' = x - v_0 t; \quad y' = y; \quad z' = z; \quad t' = t$$

$$l: \gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$l: \Delta t' = \gamma \Delta t$$

$$l: L' = \frac{L}{\gamma}$$

$$l: v'_x = \frac{v_x - v_0}{1 - \frac{v_0 v_x}{c^2}}$$

$$l: x' = \gamma(x - v_0 t); \quad y' = y; \quad z' = z; \quad t' = \gamma\left(t - \frac{v_0}{c^2} x\right)$$

$$l: \Delta x = \gamma(L' + v_0 \Delta t'); \quad \Delta t = \gamma(\Delta t' + \frac{v_0}{c^2} L')$$

$$l: p = \gamma m v$$

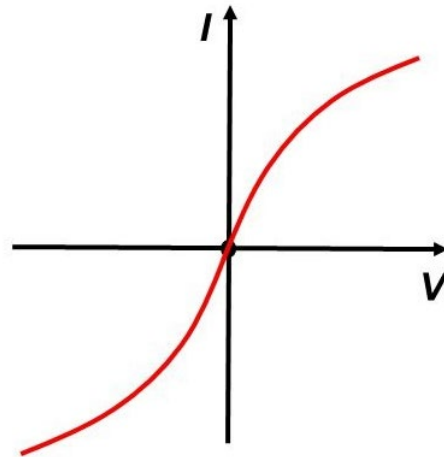
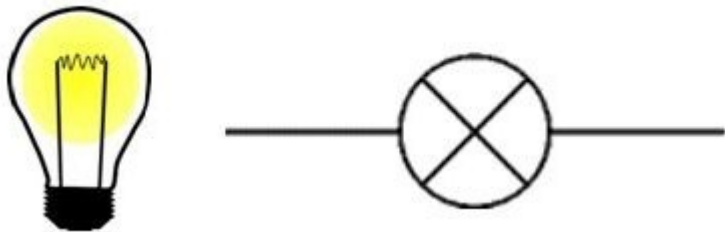
$$l: W_k = (\gamma - 1) m c^2$$

$$l: p_f = \frac{W_f}{c} = \frac{h}{\lambda}$$

$$l: W = \gamma m c^2 = c \sqrt{p^2 + m^2 c^2}$$

Linearna funkcija

- $y = kx + n$
- Ohmov zakon: $U = RI$



- gibanje s konstantno hitrostjo $s = vt$
 - primer: avtomobil se premika s konstantno hitrostjo 60 km/h. Kolikšno pot prevozi v treh urah?

Voziček na drči

- gibanje s konstantno hitrostjo
- gibanje s konstantnim pospeškom

- poskus Vernier

Kvadratna funkcija

- $y = ax^2 + bx + c$
- prosti pad (poševni met)
https://phet.colorado.edu/sims/html/projectile-motion/latest/projectile-motion_all.html
- energijski izrazi: $W_k = \frac{1}{2}mv^2$, $W_{pr} = \frac{1}{2}kx^2$
- blatni vzorec



Parabola - curek iz cevi

- $a_x = 0$ $a_y = g$
- $v_x = v_0 \cos \varphi$ $v_y = v_0 \sin \varphi - gt$
- $x = v_0 \cos \varphi t$ $y = v_0 \sin \varphi t - \frac{1}{2}gt^2$
- $t = \frac{x}{v_0 \cos \varphi}$

- $y = -\frac{g}{2v_0^2 \cos^2 \varphi} x^2 + \tan \varphi x = ax^2 + bx + c$

- $h = \frac{v_0^2 \sin^2 \varphi}{2g}$, $d = \frac{2v_0^2 \cos \varphi \sin \varphi}{g}$



Kako nastane vzorec - dve hipotezi

- enaka velikost hitrosti/naključni koti
- enaka vodoravna komponenta hitrosti, naključna navpična



Kako nastane vzorec - dve hipotezi

- enaka velikost hitrosti/naključni koti
- enaka vodoravna komponenta hitrosti, naključna navpična

- predpostavke?

- testiranje hipotez?
- teoretična obravnava in primerjava z meritvami

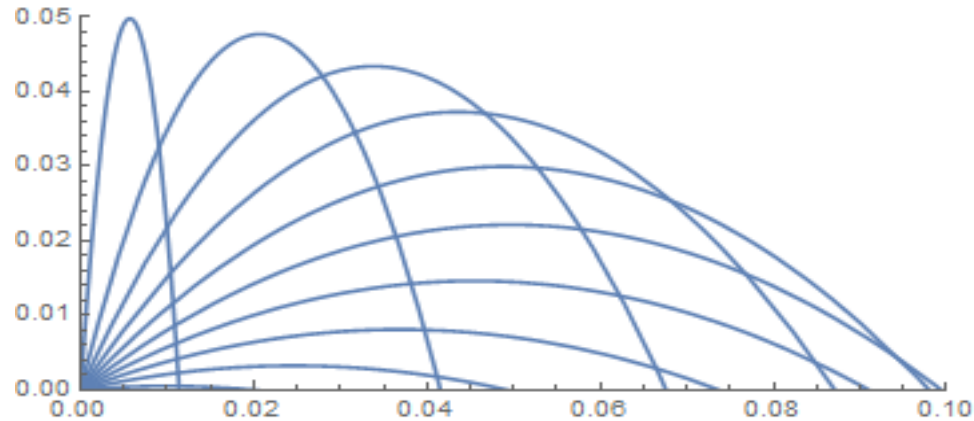
- 1. model

- $x = v_a \cos \varphi t,$

- $y = v_a \sin \varphi t - \frac{1}{2}gt^2$

- $h = \frac{v_a^2 \sin^2 \varphi}{2g}, d = \frac{2v_a^2 \cos \varphi \sin \varphi}{g}$

- $d = \sqrt{\frac{8v_a^2 h}{g} - 16h^2}$



- 2. model

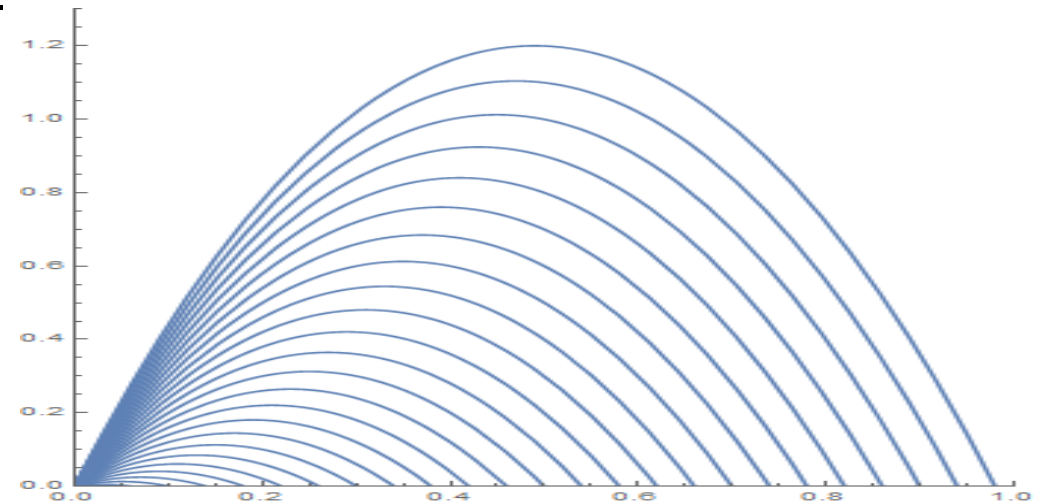
- $x = v_a t, y = v t - \frac{1}{2}gt^2,$

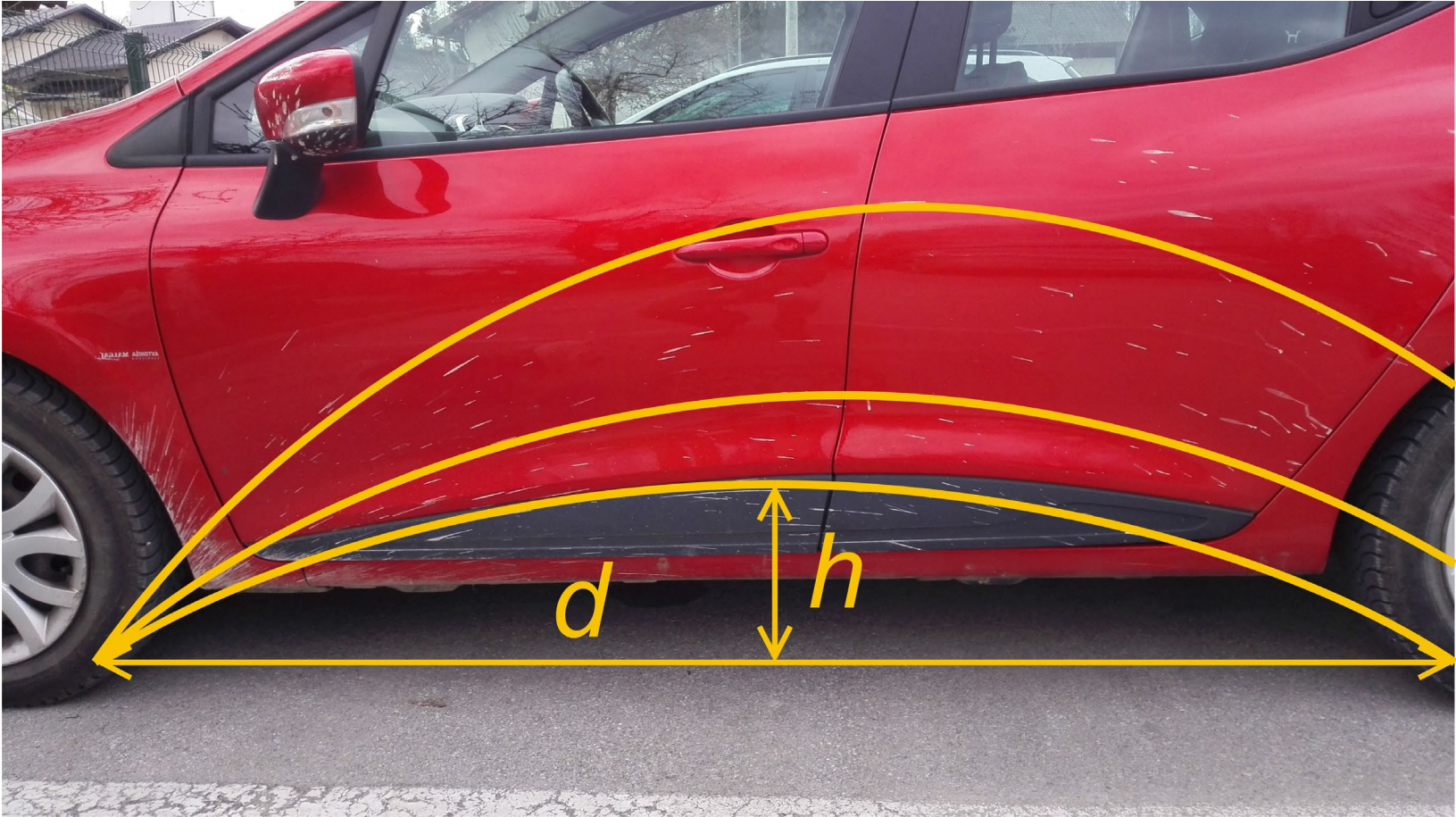
- $v_y = v - gt, v_y = 0 = v - gt_{\frac{1}{2}}, t_{\frac{1}{2}} = \frac{v}{g}$

- $h = y(t_{\frac{1}{2}}) = v \frac{v}{g} - \frac{1}{2}g \frac{v^2}{g^2} = \frac{v^2}{2g},$

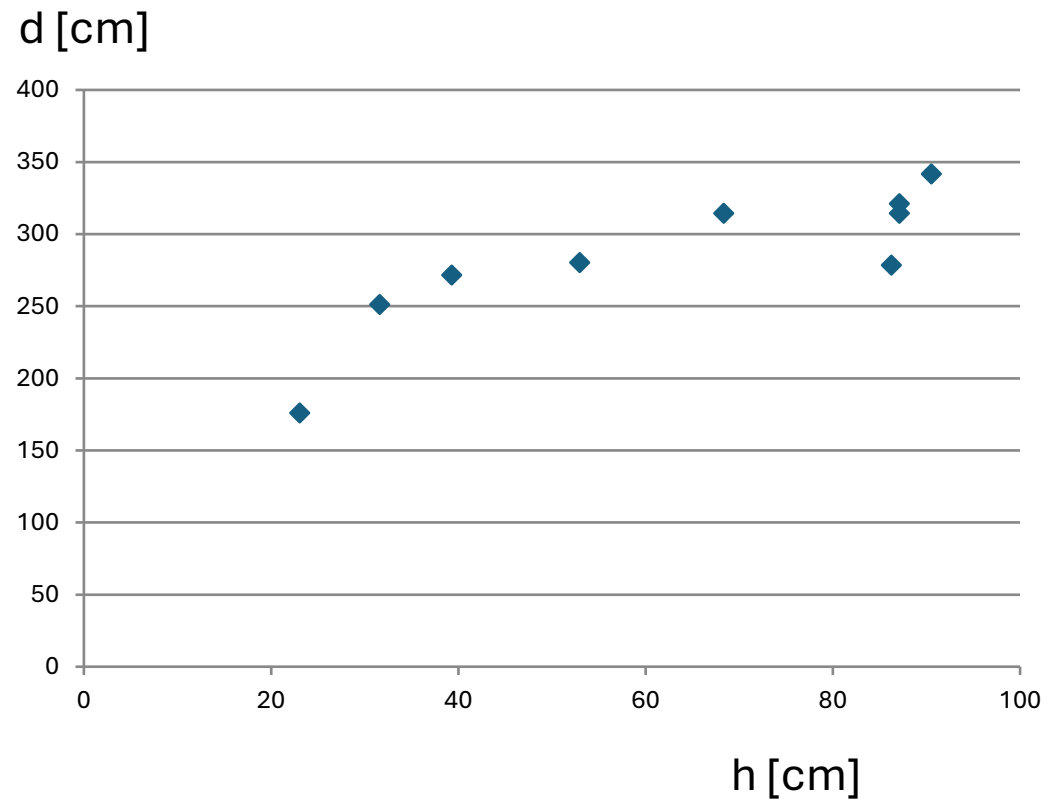
- $d = x(2t_{\frac{1}{2}}) = \frac{v_a v}{g}$

- $d = \frac{v_a \sqrt{2h}}{\sqrt{g}}$



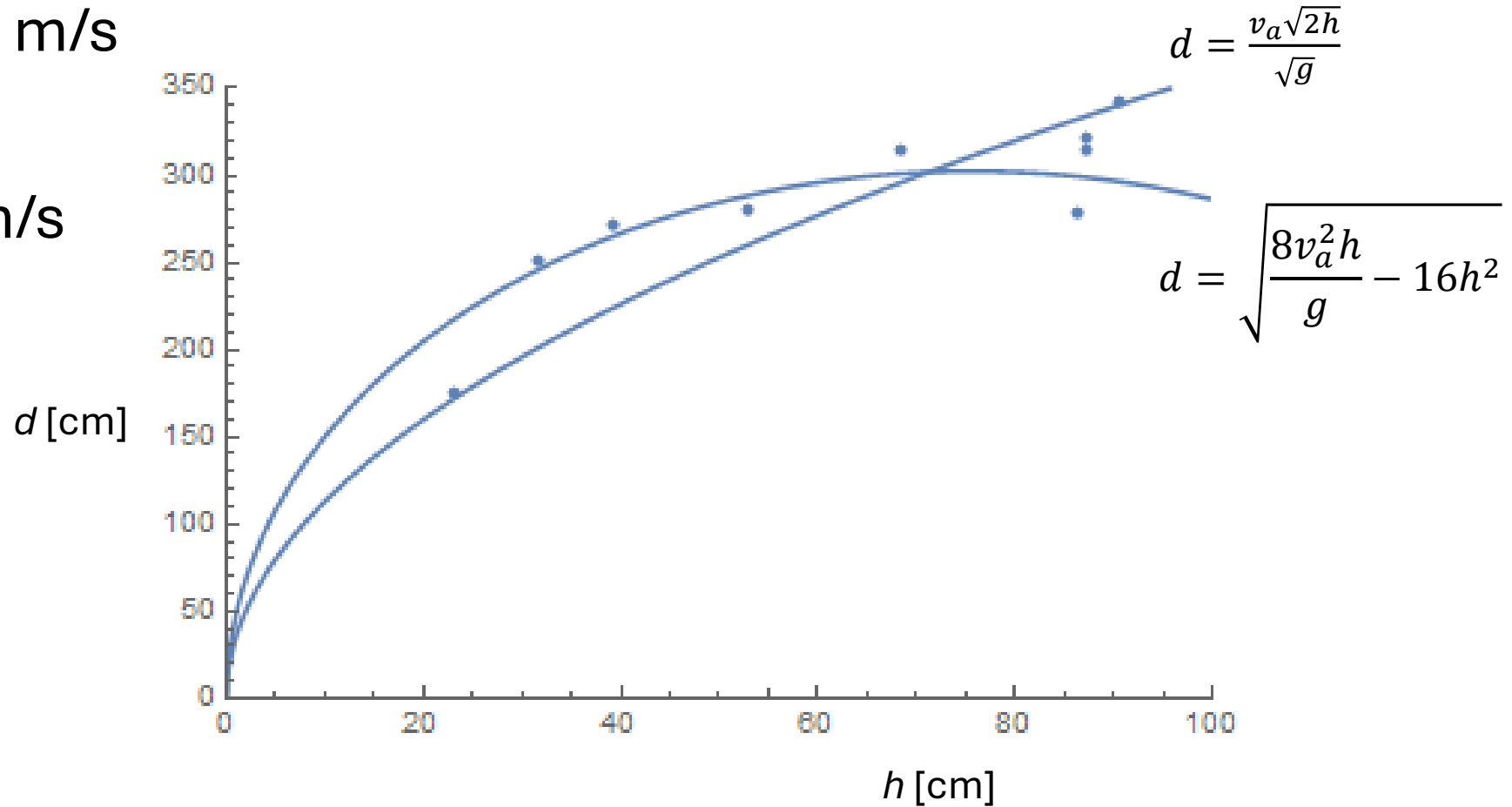


Meritve



Prilagajanje modela

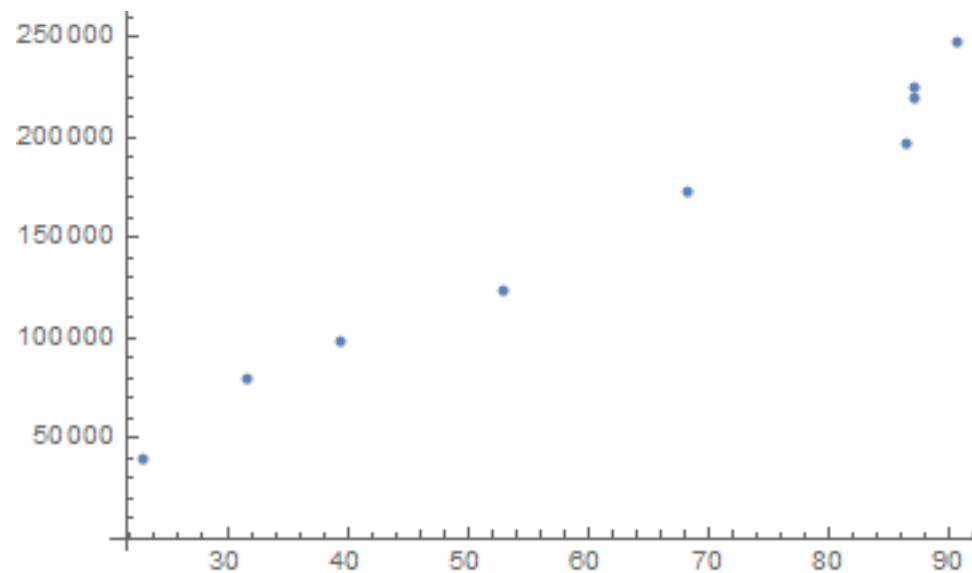
- $v_a = 5,5 \text{ m/s}$
- ali
- $v_a = 8 \text{ m/s}$



$$d^2 + 16h^2 = \frac{8v_a^2}{g} h$$

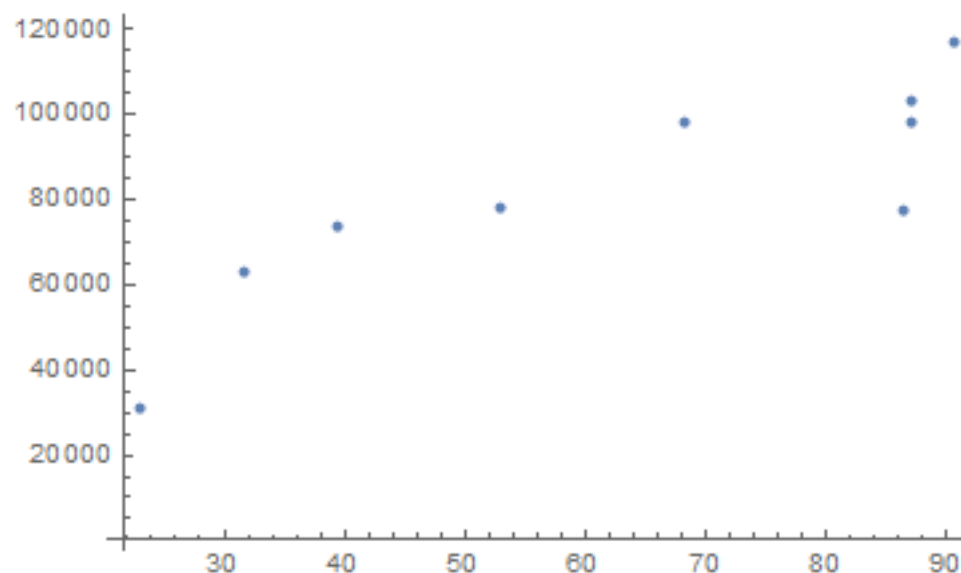
$$d^2 = \frac{2v_a^2}{g} h$$

$d^2 + 16h^2$



h

d^2



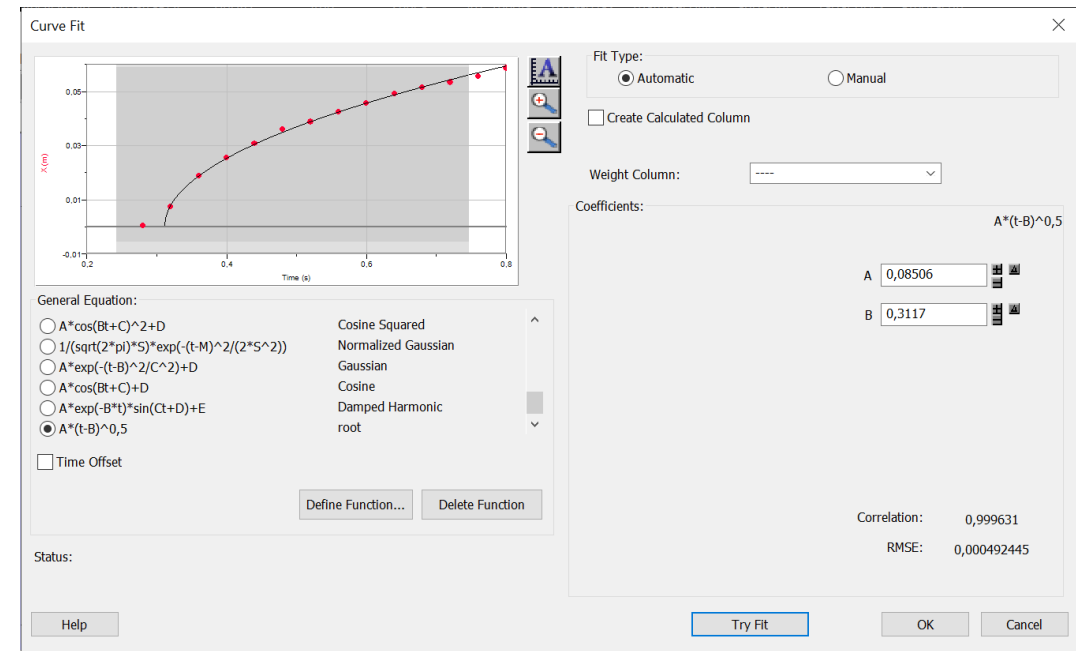
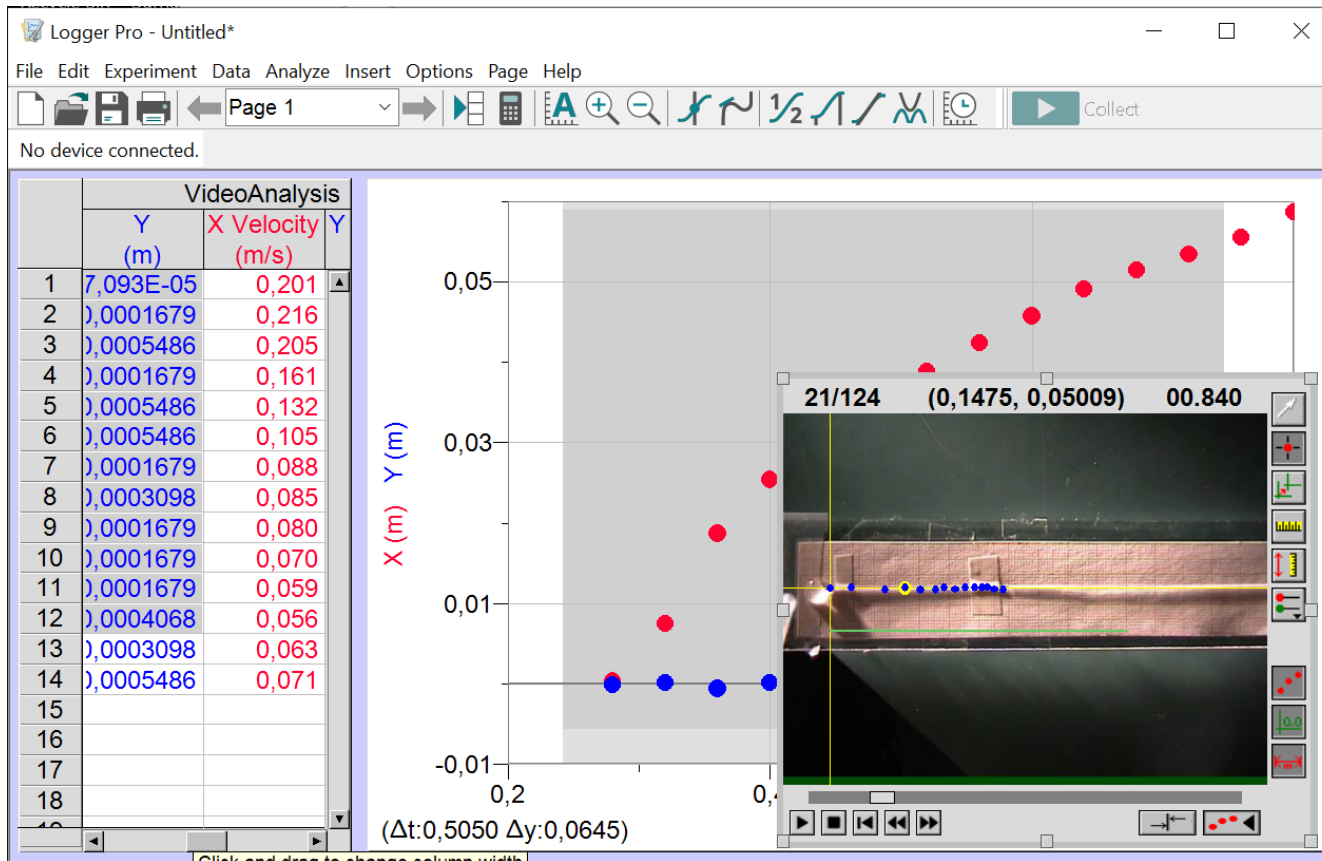
h

Korenska funkcija

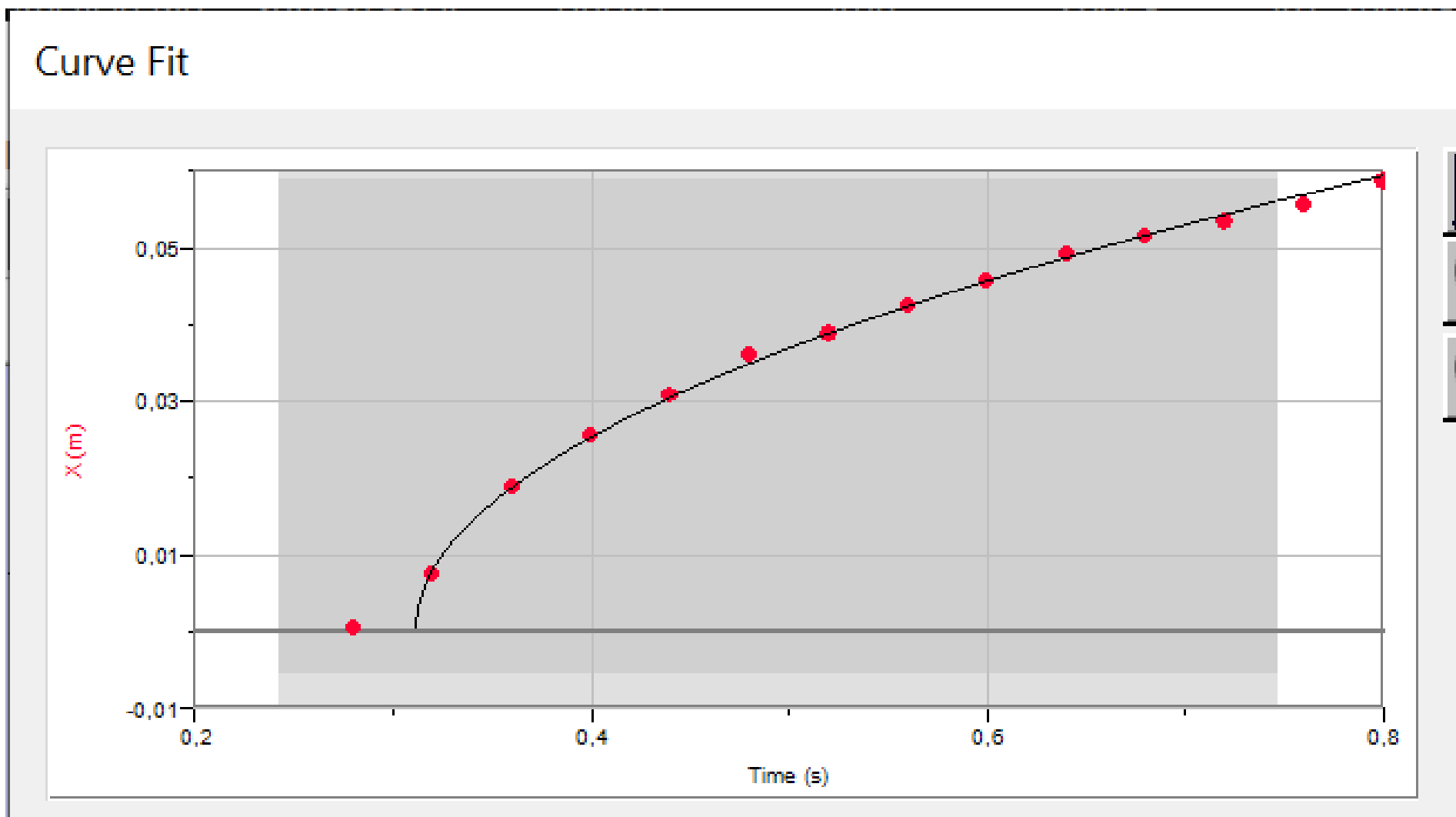
- <https://youtu.be/ZmexruMUUKg>
- analiza z LoggerPro

Korenska funkcija

- <https://youtu.be/ZmexruMUUKg>



Korenska funkcija



Korenska funkcija

- Newtonov zakon ne $F = ma$ ampak $F = \frac{dmv}{dt} = \dot{m}v + m\dot{v}$
- $\gamma 2\pi r \sin \theta - 8\pi\nu x \pi r^2 \dot{x} = \rho \pi r^2 \dot{x}^2 + \rho \pi r^2 x \ddot{x}$
- $b = ax\dot{x} + \dot{x}^2 + x\ddot{x}$
- $x = c\sqrt{t}, \dot{x} = \frac{c}{2\sqrt{t}}, \ddot{x} = -\frac{c}{4t^{3/2}}$
- $b = \frac{ac^2}{2} + \frac{c^2}{4t} - \frac{c^2}{4t}$



Pomoč UI za zglede

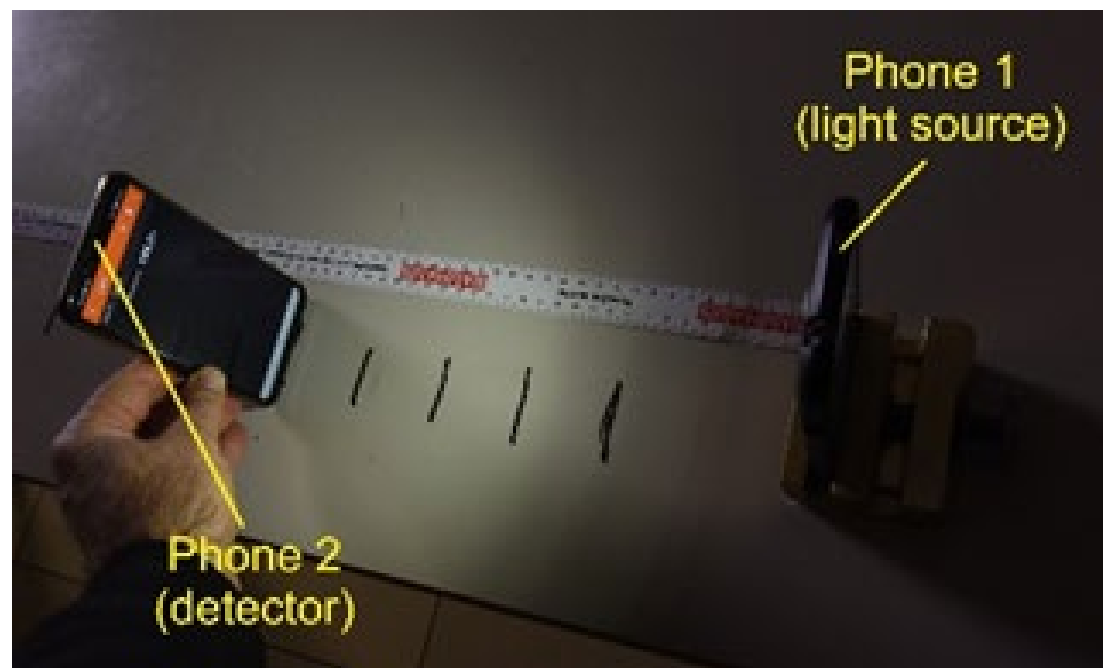
give me 10 example of square root function in physics

Sure! Here are 10 examples where square root functions are used in physics:

1. **Kinematic Equations:** Time of flight for an object under gravity, $t = \sqrt{\frac{2h}{g}}$.
2. **Wave Speed:** Speed of a wave on a string, $v = \sqrt{\frac{T}{\mu}}$ (where T is tension and μ is mass per unit length).
3. **Capacitance:** Energy stored in a capacitor, $E = \frac{1}{2}CV^2$, where $V = \sqrt{\frac{2E}{C}}$.
4. **Quantum Mechanics:** Energy levels of a hydrogen atom, $E_n = -\frac{13.6 \text{ eV}}{n^2}$, where n is an integer and involves square root calculations.
5. **Escape Velocity:** $v_e = \sqrt{\frac{2GM}{R}}$, where G is the gravitational constant, M is the mass of the planet, and R is the radius of the planet.
6. **Relativistic Time Dilation:** $t' = t\sqrt{1 - \frac{v^2}{c^2}}$, where t is the proper time, v is the velocity, and c is the speed of light.
7. **Doppler Effect:** Observed frequency, $f' = f\sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}}$.
8. **Root Mean Square Velocity:** $v_{rms} = \sqrt{\frac{3k_B T}{m}}$, where k_B is Boltzmann's constant, T is the temperature, and m is the mass of a particle.
9. **Acoustic Impedance:** $Z = \sqrt{\rho B}$, where ρ is the density of the medium and B is the bulk modulus.
10. **LCR Circuit Resonance:** Resonant frequency, $f_0 = \frac{1}{2\pi\sqrt{LC}}$, where L is inductance and C is capacitance.

Potenčna funkcija

- $y = x^n$
- za $n = -2$: Odvisnost gostote svetlobnega toka od razdalje do točkastega svetila



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Trigonometrijska funkcija

- npr. $y = \cos x$
- harmonično nihanje $x = x_0 \cos\left(2\pi \frac{t}{t_0}\right) = x_0 \cos \omega t$
 - primer: po kolikšnem času je nihalo na sredini med skrajno in ravnovesno lego?
- valovanje – težavno, funkcija dveh(!) spremenljivk $y(x, t) = y_0 \cos(kx - \omega t)$

EkspONENTNA FUNKCIJA

- $y = e^x$
- <https://youtu.be/vpVkOmWztgA>
- radioaktivni razpad $N = N_0 e^{-\lambda t}$ ali $N = N_0 2^{-\frac{t}{t_{1/2}}}$
 - primer: izrazite λ s $t_{1/2}$
- dušeno nihanje

Logaritemaska funkcija

- $y = \log x$
- glasnost $g = 10 \log_{10} \frac{j}{j_0}$ (enota je decibel, dB)
 - primer: koliko glasnejše je kričanje desetih oseb od kričanja ene?

Vključevanje matematičnih funkcij v pouk fizike

- strategije poučevanja
- medpredmetni projekti
- uporaba tehnologije in simulacij
- spodbujanje kritičnega razmišljanja in spretnosti reševanja problemov